Weighted analytic regularity and hp-FEM for the integral fractional Laplacian

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In this talk, we consider PDEs involving fractional powers of the Laplacian $(-\Delta)^s$ for $s \in (0,1)$, using the singular integral definition. Solutions to fractional PDEs are known to lose regularity near the whole boundary of the computational domain.

On polygonal domains, we establish a precise description of the regularity of solutions for analytic data in suitably weighted Sobolev spaces reflecting both the analytic nature inside the domain and the anisotropic singular behaviour near the boundary. Unlike local elliptic operators in polygons, fractional operators in polygons require not only vertex- weighted but also additionally edge-weighted spaces.

Weighted analytic regularity results of our type can be used to design exponentially convergent hp- finite element approximations on suitable anisotropic geometric meshes.

References:

[1] M. Faustmann, C. Marcati, J.M. Melenk, Ch. Schwab *Weighted analytic regularity for the integral fractional Laplacian in polygons*, arXiv:2112.08151 (2021)

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