Mixed Finite Element Method for 2nd Order Dirichlet Boundary Control Problem

Divay Garg[1]

The optimal control problems (OCPs) subjected to partial differential equations (PDEs) have numerous applications in fluid dynamics, image processing, mathematical finance etc. The objective of OCPs is to find the optimal control which minimizes/maximizes the given cost functional with certain constraints (mainly in form of PDEs) being satisfied. There are mainly two types of OCPs available in literature namely, Distributed Control Problems where the control acts on the system through an external force and Boundary Control Problems where the control acts on the system through a Dirichlet or Neumann or Robin boundary conditions. Dirichlet boundary control problems are difficult to handle due to variational difficulty.

In many applications, it is important to obtain accurate approximation of the scalar variable and its gradient simultaneously. A common way to achieve this goal is to use mixed finite element methods. The main aim of my talk is to analyze the mixed finite element method for the second order Dirichlet boundary control problem in which the control is penalized in the energy space. Mixed finite element methods have the property that they maintain the discrete conservation law at the element level. For the variational formulation, the state equation is converted to the mixed system using the mixed variational scheme for second order elliptic equations and then the continuous optimality system is derived. In order to discretize the continuous optimality system, the lowest order Raviart-Thomas space is used to numerically approximate the state and costate variables whereas the continuous piece-wise linear finite element space is used for the discretization of control. Based on this formulation, the optimal order a priori error estimates for the control in the energy norm and L_2-norm is derived. The reliability and the efficiency of proposed a posteriori error estimator is also discussed using the Helmholtz decomposition. Finally, several numerical experiments are presented to confirm the theoretical findings.

References:

[1] S. Chowdhury, T. Gudi, and A. K. Nandakumaran. Math. Comp., 86:1103-1126, 2017.

[2] W. Gong and N. Yan. SIAM J. Control Optim., 49:984–1014, 2011.

[3] F. Trolzsch. Optimale Steuerung Partieller Differentialgleichungen. Viweg, Cambridge University Press, 2005.

[1]Indian Institute of Technology Delhi
maz188446@maths.iitd.ac.in