Finite element methods respecting the discrete maximum principle for convection-diffusion equations I

Volker John[1] Gabriel R. Barrenechea[2] Petr Knobloch[3]

Convection-diffusion-reaction equations model the conservation of scalar quantities. From the analytic point of view, solution of these equations satisfy under certain conditions maximum principles, which represent physical bounds of the solution. That the same bounds are respected by numerical approximations of the solution is often of utmost importance in practice. The mathematical formulation of this property, which contributes to the physical consistency of a method, is called Discrete Maximum Principle (DMP). In many applications, convection dominates diffusion by several orders of magnitude. It is well known that standard discretizations typically do not satisfy the DMP in this convection-dominated regime. In fact, in this case,it turns out to be a challenging problem to construct discretizations that, on the one hand, respect the DMP and, on the other hand, compute accurate solutions.

This talk starts to presents a survey on finite element methods, with a main focus on the convection-dominated regime, that satisfy a local or a global DMP. The concepts of the underlying numerical analysis are explained. Some linear discretizations with P_1 finite elements satisfying DMPs are described. Linear discretizations for other finite elements are discussed briefly.

Part II of this talk will be presented by Petr Knobloch. This topic is joint work with Gabriel R. Barrenechea and Petr Knobloch.

References:

[1] https://arxiv.org/abs/2204.07480

[1]Weierstrass Institute for Applied Analysis and Stochastics, Weierstrass Institute for Applied Analysis and Stochastics
john@wias-berlin.de

[2]Department of Mathematics and Statistics, University of Strathclyde, 26 Richmond Street, Glasgow G1 1XH, Scotland
gabriel.barrenechea@strath.ac.uk

[3]Department of Numerical Mathematics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 18675 Praha 8, Czech Republic
knobloch@karlin.mff.cuni.cz