Using structured multigrid methods in unstructured FEM solvers

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Multigrid methods have been developed for both structured and unstructured grids. Due to their purely algebraic construction of coarse levels, algebraic multigrid (AMG) methods are particularly appealing for unstructured grid scenarios. Yet, they come with considerable setup cost due to indirect indexing into matrices defined on unstructured grids as well as communication on parallel machines, since many operations (foremost the Galerkin product RAP) require communication among data structures defined on the entire mesh. In contrast, hierarchical hybrid grids (HHGs) apply local grid refinement to benefit from structured grid performance, however cannot be interfaced with mature application codes in a straightforward manner [1].

In [2], we propose a non-invasive multigrid method that combines the idea of AMG and HHG: while grid structure in some localized parts of the mesh allows for efficient structured grid algorithms, a hierarchy of coarse levels is formed in a purely algebraic fashion. This solver, named Region MG, can easily be used with existing application codes and allows to leverage structured multigrid components for partially structured meshes without re-implementing the discretization and matrix assembly procedure. Based on minimal user-provided meshing information, the multigrid hierarchy can be defined purely locally. Communication among processes is only required during an initial pre-processing of the input matrix as well as after restriction to a coarser level. Foremost, the Galerkin product RAP can be formed without any parallel communication.

In this presentation, we will develop a mathematical framework underpinning the proposed Region MG method for partially structured grids. We will show its equivalence to established AMG solvers as well as its performance benefits. We will apply Region MG to hypersonic flow problems and demonstrate its superiority over more traditional AMG preconditioners.

References:

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