A priori and a posteriori error estimates for the Deep Ritz method applied to the Laplace and Stokes problem

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We analyze neural network solutions to partial differential equations obtained with Physics Informed Neural Networks. In particular, we apply tools of classical finite element error analysis to obtain conclusions about the error of the Deep Ritz method applied to the Laplace and the Stokes equations. Further, we develop an a posteriori error estimator for neural network approximations of partial differential equations. The proposed approach is based on the dual weighted residual estimator. It is destined to serve as a stopping criterion that guarantees the accuracy of the solution independently of the design of the neural network training. The result is equipped with computational examples for Laplace and Stokes problems.

References:

[1] https://arxiv.org/abs/2107.11035

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