On a deep neural network algorithm for solving backward heat conduction problems

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In recent years, due to wide applicability, straightforward implementation, and great performance in practice, there is an interest in applying the artificial neural network (ANN), or simply neural network (NN), strategy to solve numerous inverse problems in many disciplines. Partial differential equations (PDEs) used to model real-world problems are one category that researchers try to solve by employing neural networks to eliminate the loss of numerical discretization and avoid mesh generation. Recently, the idea of solving forward and inverse PDEs with deep neural networks was proposed in by Raissi et al. These networks are referred to as physics-informed neural networks (PINNs). The reason behind the success of PINNs is relying on well-developed tools such as automatic differentiation for dealing with partial differential terms. Moreover, neural network models are good approximators for high-dimensional functions that can be trained efficiently in most cases. Mostajeran and Mokhtari applied PINNs to solve backward heat conduction problems (BHCPs), which have been long-standing computational challenges due to being ill-posed. The primary benefit of this method is overcoming the complexity of the domain with various boundary conditions. Without needing any discretization of the time or space, the algorithm can retrieve the unknown solution on the domain if given a sufficient number of sampled training data. This revolutionary strategy can efficiently and accurately extract the pattern of the solutions even when the noise corruption of up to ten percent is imposed on input data. Moreover, when the final time is increased further, this approach is efficient in recovering the data at the initial time, which accentuates the method's robustness. We also show that there exists a neural network such that the corresponding loss function converges to zero. Furthermore, we prove that this neural network strongly converges to the final solution on the domain and weakly converges to the solution of the BHCP over its domain at each time.

References:

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