Formation of wrinkles in bi-layer systems

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We model the formation of wrinkles of an elastic substrate coated with a thin film. The elastic substrate is first stretched, then the film is attached to a part of the substrate boundary in the deformed state. Once the the external force is released, wrinkles form due to the stress mismatch between the two materials. The elastic substrate is modeled using a hyperelastic, homogeneous and isotropic material. The film is modeled using a geometrically exact Cosserat shell. The resulting deformation and microrotation (φ, R) are a minimizing pair of the combined energy functional

$$J(\varphi, R) = \int_\Omega W_{\mathsf{bulk}}(\nabla\varphi)\, dV + \int_{\Gamma_c} W_{\mathsf{coss}}(\nabla\varphi_{|\Gamma_c}, R)\, dS$$

in the admissible set

$$\mathcal{A} = \Big\{ (\varphi, R) \in W^{1,q}(\Omega, \mathbb{R}^3) \times H^1(\Gamma_c, \mathrm{SO(3)}) \;\Big|\; \varphi \text{ is a deformation function, } (\varphi, R) \text{ fullfill the Dirichlet boundary conditions} \Big\}$$

with $q > 3$. We discretize the problem using Lagrange finite elements for the substrate displacement and geodesic finite elements for the microrotation field. Geodesic finite elements are a generalization of standard finite elements to spaces of functions mapping to a Riemannian manifold. We prove the existence of solutions of the discrete coupled model.
Numerical experiments show that we can efficiently reproduce wrinkling patterns of coupled systems. Our approach works as well for more complex scenarios like multi-layer systems involving different stress-free configurations.

References:

[1] https://arxiv.org/pdf/1412.3668.pdf

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