Error analysis for a moving boundary problem describing the penetration of a diffusant concentration into rubbers

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We propose a moving-boundary scenario to model the penetration of diffusants into rubbers. Immobilizing the moving boundary by using the well-known Landau transformation transforms the original governing equations into new equations posed in a fixed domain. We solve the transformed equations by the finite element method. Numerical simulation results show that the computed penetration depths of the diffusant concentration are within the range of experimental measurements. To have trust in the obtained simulation results, we perform the numerical analysis for our setting. Initially, we study semi-discrete finite element approximations of the corresponding weak solutions. We prove both a priori and a posteriori error estimates for the mass concentration of the diffusants, and respectively, for the a priori unknown position of the moving boundary. Finally, we present a fully discrete scheme for the numerical approximation of model equations. Our scheme is based on the Galerkin finite element method for the space discretization combined with the backward Euler method for time discretization. In addition to proving the existence and uniqueness of a solution to the fully discrete problem, we also derive a priori error estimates for the mass concentration of the diffusants, and respectively, for the position of the moving boundary. Our numerical illustrations verify the obtained theoretical order of convergence in physical parameter regimes.

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