Finite element error estimates for PDEs with irregular Dirichlet boundary data using boundary concentrated meshes

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This talk is concerned with finite element error estimates for PDEs with inhomogeneous Dirichlet boundary data in convex polygonal domains. The Dirichlet boundary data are assumed to be irregular such that the solution of the PDE does not belong to $H^2(\Omega)$ but only to $H^t(\Omega)$ for some $t \in (1,2)$. As a consequence, a discretization of the PDE with linear finite elements exhibits a reduced rate in $L^2(\Omega)$ and $H^1(\Omega)$. In order to restore the best possible convergence rate we propose and analyze in detail the usage of boundary concentrated meshes. These meshes are gradually refined towards the whole boundary. The corresponding grading parameter does not only depend on the regularity of the Dirichlet boundary datum but also on the norm, which is used to measure the error. That means, different grading conditions are necessary for the error in $L^2(\Omega)$ and $H^1(\Omega)$. In numerical experiments we confirm the behavior predicted by our analysis.

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