A local discontinuous Galerkin method for the subdiffusion inverse source problem with a weakly singular solution

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In this talk, the aim is to determine the source term in an inverse problem corresponding to the following time-fractional diffusion, subdiffusion, equation

$$\begin{cases} \mathcal{D}_t^\alpha u - \Delta u = f(x)p(t), & (x,t) \in \Omega \times (0,T], \\ u(\cdot,t)|_{\partial\Omega} = g(t), \quad t \in [0,T]; & u(x,0) = u_0(x), \quad x \in \Omega, \end{cases}$$

where u and f or p are unknown functions, and $\mathcal{D}_t^\alpha$ stands for the Caputo fractional derivative of order $\alpha \in (0,1)$, i.e. $\mathcal{D}_t^\alpha u(\cdot,t) = (1/\Gamma(1-\alpha)) \int_0^t (t-s)^{-\alpha}\, , u_t(\cdot,s)\, , \mathrm{d}s$. To solve the inverse problem, we need one of the following over-specified conditions

$$\begin{aligned} u(x,T) &= h_1(x), & x &\in \Omega, \\ u(x^*,t) &= h_2(t), & t &\in [0,T], \quad x^* \in \Omega. \end{aligned}$$

The time-fractional diffusion problems taking into consideration Caputo fractional derivative usually have singular solutions near the initial time, i.e. $t = 0$. In this case, numerical methods generated on the uniform mesh break down in facing the problem and adaptive mesh refinement is needed near the initial time to hold the convergence order. Yeganeh et al. proposed a method based on the well-known L1 formula and the local discontinuous Galerkin (LDG) scheme to determine the space-dependent source term. Here, we consider both space- and time-dependent source determination in the inverse problem with a weakly singular solution.
We extend a method based on the L1-2-3 formula, proposed by Mokhtari and Mostajeran, for approximating the Caputo derivative and a local discontinuous Galerkin method in the spatial direction. The L1-2-3 approximation is used on a mesh that is graded in time, taking into account the initial singularity of the solution. It is indicated that the convergence order of the proposed graded L1-2-3 LDG method is $\mathcal{O}(N^{\min\{r,3\}})$ while the convergence order of methods used uniform meshes is reduced. Finally, numerical results show the effect of the adaptively graded mesh on the convergence order of the proposed method.

References:

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