Numerical Analysis of optimal control problem governed by transient Stokes equations with state constraints pointwise in time

Boris Vexler[1] Dmitriy Leykekhman[2]

We consider the following optimal control problem:

$$\text{Minimize } \frac{1}{2}\int_0^T \int_\Omega (u(t,x) - u_d(t,x))^2 \, dx \, dt + \frac{\alpha}{2}\int_0^T \int_\Omega q(t,x)^2 \, dx \, dt$$

subject to transient Stokes equations

$$\begin{aligned} \partial_t u - \Delta u + \nabla p &= q && \text{in } (0,T) \times \Omega \\ \nabla \cdot u &= 0 && \text{in } (0,T) \times \Omega \\ u &= 0 && \text{on } (0,T) \times \partial\Omega \\ u(0) &= u_0 && \text{in } \Omega \end{aligned}$$

and to state constraints formulated pointwise in time, i.e.

$$\int_\Omega u(t,x) \cdot w(x) \, dx \leq b \quad \text{for all } t \in [0,T]$$

with a given $u_d \in L^2(I; L^2(\Omega)^d)$ and $w \in L^2(\Omega)^d$. The domain $\Omega \subset \mathbb{R}^d$, $d = 2,3$ is assumed to be polygonal/polyhedral and convex.

The optimality system for this problem involves a Lagrange multiplies from the space of regular Borel measures $\mu \in \mathcal{M}([0,T])$, which affects the regularity of the solution. We discretize the problem with inf-sup stable finite elements in space and with a discontinuous Galerkin method in time. For this discretization we provide quasi-optimal error estimates for the state and the control. The analysis is based on recently established error estimates for the Stokes system [1].

References:

[1] N. Behringer, D. Leykekhman, B. Vexler: Fully discrete best approximation type estimates in $L^\infty(I; L^2(\Omega)^d)$ for finite element discretizations of the transient Stokes equations, *IMA Journal of Numerical Analysis* (2022).

[1]Technical University of Munich
vexler@ma.tum.de

[2]University of Connecticut
dmitriy.leykekhman@uconn.edu