Fully Discrete Error Estimates for Finite Element Discretizations of the Instationary 2D Navier-Stokes Equations

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We consider the instationary Navier-Stokes equations with homogeneous Dirichlet boundary conditions in a 2D domain Ω on a bounded time interval I. The equations are discretized in space by inf-sup stable pairs of finite element spaces, and in time via the discontinuous Galerkin method. This choice allows for a variational formulation of the discretized equations, which facilitates definition of dual equations, particularly useful for error estimates and the analysis of optimal control problems. We will derive an estimate of the error in the $L^\infty(I;L^2(\Omega))$ norm, which is of a best approximation-type and thus has optimal orders of convergence w.r.t. the spacial discretization parameter h and the time discretization parameter k. Under H^2 regularity assumptions and a right hand side in $L^\infty(I;L^2(\Omega))$ it is possible to relate this error estimate for the nonlinear Navier-Stokes equations to the corresponding error estimate for the linear Stokes equations, which was shown by Behringer, Vexler and Leykekhman (2022). Since the energy estimate for the solution to the Navier-Stokes equations provides a bound for the $L^2(I;H^1(\Omega))$- and $L^\infty(I;L^2(\Omega))$ norms in a combined fashion, many error estimates treat the error in these two norms jointly. This results in a suboptimal rate of convergence with respect to the spacial discretization parameter h, due to the $H^1(\Omega)$ norm contribution. Since we estimate the error individually, we can make full use of the $L^2(\Omega)$ norm, allowing for an improved estimate.

References:

[1] https://doi.org/10.1093/imanum/drac009

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