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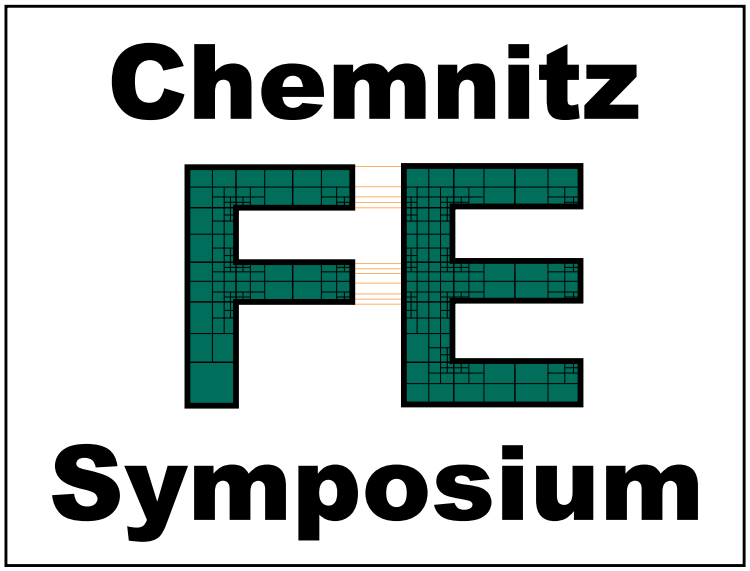
Chemnitz FE-Symposium 2023 on tour



Programme

Collection of abstracts

List of participants

Schloß Seggau, September 11 - 13, 2023

Scientific Topics:

The symposium is devoted to all aspects of finite elements and other computer-based methods for solving partial differential equations.

The topics include (but are not limited to):

- Scientific Computing,
- Mechanics/Applications,
- Inverse Problems,
- Optimization with PDEs,
- Uncertainty Quantification.

This year we particularly encourage talks on:

- Scientific Computing,
- Solvers,
- Discontinuous Galerkin.

Invited Speakers:

Martin Gander (Université de Genève)

Barbara Kaltenbacher (Universität Klagenfurt)

Yvan Notay (Université Libre de Bruxelles)

David E. Keyes (King Abdullah University of Science and Technology)

Scientific Committee:

Th. Apel (München), F. Bertrand (Chemnitz), S. Beuchler (Hannover), O. Ernst (Chemnitz), G. Haase (Graz), H. Harbrecht (Basel), R. Herzog (Heidelberg), M. Jung (Dresden), U. Langer (Linz), A. Meyer (Chemnitz), O. Rheinbach (Freiberg), A. Rösch (Duisburg-Essen), O. Steinbach (Graz), M. Stoll (Chemnitz), M. Winkler (Chemnitz)

Organising Committee:

G. Haase, M. Winkler, F. Caforio, T. Grandits
https://mathematik.uni-graz.at

Internet Access

Schloß Seggau offers free internet access. Details will be obtained at the reception.
The Wi-Fi in the seminar rooms can be accessed using the password *seminar18*.

Food

The conference fee includes:

- Lunch around 12:00 on all three days of the symposium
- Tea and coffee during breaks
- Dinner on Monday and conference dinner on Tuesday.

For participants staying at the Schloß Seggau there is a breakfast buffet from 7:00 up to 10:00.

Conference Dinner

The conference dinner will start on Tuesday at 18:30 at the dining hall in Schloß Seggau.

Excursion to Kogelberg

The excursion will take place on Tuesday. We will meet at 14:00 in front of the conference venue.
We will hike through the vinyards to the hill Kreuzkogel (6.5km and 280m difference in altitude, 2 hours). Sports shoes or good walking shoes are suggested. Further information on the route can be obtained using the QR-code, or link:

https://tinyurl.com/2p9arezs

Programme

Programme for Monday, September 11, 2023

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Collection of Abstracts

Nonlinear Preconditioning for Implicit Solution of Discretized PDEs

David Keyes[1]

Abstract Nonlinear preconditioning refers to transforming a nonlinear algebraic system to a form for which Newton-type algorithms have improved success through quicker advance to the domain of quadratic convergence. We place these methods, which go back at least as far as the Additive Schwarz Preconditioned Inexact Newton (ASPIN, 2002), in the context of a proliferation distinguished by being left- or right-sided, multiplicative or additive, non-overlapping or overlapping, and partitioned by field, subdomain, or other criteria. We present the Nonlinear Elimination Preconditioned Inexact Newton (NEPIN, 2021), which is based on a heuristic *bad/good* heuristic splitting of equations and corresponding degrees of freedom. We augment basic forms of nonlinear preconditioning with three features of practical interest: a cascadic identification of the *bad* discrete equation set, an adaptive switchover to ordinary Newton as the domain of convergence is approached, and error bounds on output functionals of the solution. Various nonlinearly stiff algebraic and model PDE problems are considered for insight and we illustrate performance advantage and scaling potential on challenging two-phase flows in porous media.

[1]King Abdullah University of Science and Technology
david.keyes@kaust.edu.sa

Notes on session “Solvers”

Numerical approximation of nonlinear Schrödinger equations by localized orthogonal decomposition

Christian Döding[1] Patrick Henning[2] Johan Wärnegard[3]

The Gross-Pitaevskii equation (GPE) is a nonlinear Schrödinger equation which is used in quantum physics to model the dynamics of Bose-Einstein condensates (BECs). It is well known that this equation has important time invariants such as the total energy of the system. Preserving the energy under numerical discretization can be of great significance in many practical situations. In this talk we consider numerical approximations of the GPE based on multiscale approaches. To be more precise, we choose a generalized finite element space which is based on the localized orthogonal decomposition method and which allows to capture the energy with high accuracy. Paired with energy-preserving time integrators we demonstrate how such an approach can lead to an efficient solver for the GPE and thus for the simulation of the dynamics of BECs on larger time scales.

[1]Ruhr-University Bochum, Department of Mathematics
christian.doeding@rub.de

[2]Ruhr-University Bochum, Department of Mathematics
patrick.henning@rub.de

[3]Columbia University, Department of Applied Physics and Applied Mathematics
jpw2176@columbia.edu

Optimizing multigrid smoothers for high-order matrix-free FEM computations

Peter Munch[1] Martin Kronbichler[2]

Multigrid methods are among the most competitive solvers for linear systems arising upon discretization of second-order partial differential equations, which occur as subproblems in many application fields in computational science. Within the multigrid algorithm, the most crucial component is usually the smoother, which aims to reduce the high-frequency content in the iteration errors. The observed performance of a smoother depends on the error reduction rate, the possibility for high-performance implementations, and the properties of the underlying hardware itself.

In this presentation, we investigate, in the context of high-order matrix-free FEM computations, point-Jacobi preconditioners and additive Schwarz methods (ASM) based on the fast diagonalization method (FDM) defined on overlapping cell-centered and vertex-star patches. We embed both preconditioners into classical relaxation schemes as well as a Chebyshev iteration and use them as smoothers in the context of p-multigrid. We present novel, highly optimized implementations, which leverage the caches of modern processors by interleaving the work done on the cells in the finite-element discretization and vector operations. We have developed an infrastructure that allows such interleaving in the context of preconditioned conjugate gradient methods [4] and made the implementation freely available via the library deal.II. Here, we show a new application case of this infrastructure.

We conclude our presentation by embedding the developed smoothers into a multigrid scheme to solve Poisson problems on anisotropic meshes. We summarize the results of extensive parameter studies, where we investigate the influence of the number of smoothing steps, the type of decreasing the polynomial degree, the type of preconditioner, the type of Chebyshev polynomials (first or fourth kind), and the type of V-cycle (one- or two-sided). Our results indicate that ASM with FDM, when using the proposed optimizations, can outperform point- Jacobi smoothers on modern CPU-based hardware, especially for meshes with lower-quality and anisotropic elements.

References:

[1] M. Kronbichler and K. Kormann, 2012

[2] RE. Lynch et. al, 1964

[3] JW. Lottes and PF. Fischer, 2005

[4] M. Kronbichler et. al., 2022

[5] M. Phillips and PF. Fischer, 2022

[1]High-Performance Scientific Computing, Institute of Mathematics, University of Augsburg
peter.muench@uni-a.de

[2]High-Performance Scientific Computing, Institute of Mathematics, University of Augsburg
martin.kronbichler@uni-a.de

High order biorthogonal basis functions

Tim Haubold[1] Sven Beuchler[2] Joachim Schöberl[3]

Dual basis function are used to define (high order) interpolation operators. We consider high order basis functions based on Legendre and Jacobi polynomials. In this case the dual functions are biorthogonal to our chosen basis. In this talk we will consider this property in $2D$ and $3D$. Moreover we derive biorthogonal basis functions for vector valued functions in $H(\mathrm{Curl})$.

[1]Leibniz University Hannover, Institute for Applied Mathematics
haubold@ifam.uni-hannover.de

[2]Leibniz University Hannover, Institute for Applied Mathematics
beuchler@ifam.uni-hannover.de

[3]TU Wien, Institute of Analysis and Scientific Computing
joachim.schoeberl@tuwien.ac.at

IMEX methods for incompressible flows with variable viscosity

Douglas Ramalho Queiroz Pacheco[1] Ernesto Castillo[2] Gabriel Barrenechea[3]

In implicit-explicit (IMEX) temporal discretisations of the Navier-Stokes equations, the viscous term is kept implicit, while convection is treated either explicitly or semi- implicitly. Such methods are very popular, as they simplify implementation, improve computational efficiency and retain good numerical stability. For flows with non-constant viscosity, as arising in various practical applications, it can be attractive to treat also the viscous term in a semi-implicit manner. That allows us, for instance, to use simpler solvers by avoiding additional terms that would otherwise couple the velocity components. In this talk, we consider different IMEX treatments of the viscous term and discuss the implications of such approaches. This includes deriving, for first-order schemes in time, stability estimates that do not incur a CFL-like condition. Both monolithic and fractional-step methods are considered, with numerical examples corroborating our theory.

[1]Norwegian University of Science and Technology, Department of Mathematical Sciences
douglas.r.q.pacheco@ntnu.no

[2]Universidad de Santiago de Chile
ernesto.castillode@usach.cl

[3]University of Strathclyde
gabriel.barrenechea@strath.ac.uk

Notes on session “Solvers”

Reduced order modeling of time-dependent generalized Newtonian fluid flows

Ernesto Castillo[1]

This work numerically evaluates the accuracy and performance of a stabilized finite element Reduced Order Modelling (ROM) approach designed to simulate time-dependent generalized Newtonian fluid flows. The method estimates off-trained parametric scenarios not included in the training data set composing the ROM basis and can adopt arbitrary values from other specific fluid and flow conditions. Also, a mesh-based hyper-reduction technique is included. The numerical testing includes approximating well- established benchmark solutions of shear-thinning and shear-thickening fluid flows to demonstrate the method's robustness. Furthermore, the application of the method in two engineering problems related to hemodynamic and conjugate thermally coupled flows is presented. Numerical results evidence the method's capability, accuracy, and performance to approximate complex flow conditions of generalized Newtonian fluids.

This work is partially founded by ANID Chile through the project FONDECYT 1210156.

[1]University of Santiago de Chile
ernesto.castillode@usach.cl

H(div)-conforming dGFEM for turbulent incompressible wall-bounded Navier-Stokes flows

Gert Lube[1]

Three basic problems are relevant for the numerical simulation of turbulent incompressible Navier-Stokes flows: i) "curse of resolution" in space-time for direct numerical simulation (DNS) which critically depends on the Reynolds number, ii) regularity and intermittency of the solution in space-time, iii) occurence of anomalous diffusion in the inviscid limit stemming from the action of fluctuations via the Reynolds stress tensor. These problems call for good solutions w.r.t. computational ressources. We discuss some variants for the numerical modelling of turbulent wall-bounded flows.

H(div)-conforming discontinuous Galerkin-FEM ensure pointwise divergence-free discrete velocities together with pressure-robustness, convection semi-robustness and structure preservation. Moreover they allow a weak imposition of the tangential velocities at a wall and act as implicit large-eddy simulation (ILES). This will be demonstrated for the basic channel flow at bulk Reynolds numbers of 10^3 ... 10^5 for the mean streamwise velocity component and the averaged Reynolds stress tensor on slightly anisotropic meshes with less than 10^6 unknowns, see Schroeder, PhD. Thesis, Goettingen 2019, as opposed to DNS results with up to 10^9 unknowns.

For very large Reynolds numbers, Hoffman et al. in J. Math. Fluid Mech. (2016) replace the no-slip condition by a Navier-with-slip/friction condition. For a friction coefficient tending to zero, one obtains in the inviscid limit a slip condition. In aviation applications they show that a relatively coarse resolution (with up to 10^6 ... 10^7 nodes) is sufficient to obtain reasonable averaged values of lift and drag. We discuss and argue why such approach can be questionable for very large Reynolds numbers. A proper combination of H(div)-conforming dGFEM with Navier-with-slip/friction condition at the wall deserves a stronger consideration. In the inviscid limi, such method with appropriate stabilization can cope with anomalous diffusion, e.g. for the inviscid Taylor-Green vortex, see Fehn, PhD. Thesis, TU Munich 2021.

[1]Georg-August University Goettingen, Dep. Math. and Comput. Sc.
lube@math.uni-goettingen.de

Higher order discontinuous Galerkin methods in time and pressure-robust finite element discretizations applied to time-dependent Stokes problems

Gunar Matthies[1] Naveed Ahmed[2] Simon Becher[3]

We analyze finite element discretizations of the time-dependent Stokes equations that are based on discontinuous Galerkin time stepping schemes in combination with pressure-robust inf-sup stable finite element methods in space. The pressure-robustness enables error estimates for the velocity that are completely independent of the pressure. We prove optimal convergence orders in space and time for both velocity and pressure. Moreover, a cheap postprocessing allows to improve the temporal accuracy of the velocity, again with error constants independent of the pressure. Numerical examples illustrate our theoretical findings.

[1]Technische Universität Dresden, Institut für Numerische Mathematik
gunar.matthies@tu-dresden.de

[2]Gulf University for Science and Technology, Mubarak Al-Abdullah Area/West Mishref, Kuwait
Ahmed.N@gust.edu.kw

[3]Technische Universität Dresden, Institut für Numerische Mathematik
simon.becher@tu-dresden.de

An Optimally Convergent Convection-Stabilized Taylor–Hood Finite Element Method for the Oseen Equations

Marwa Zainelabdeen[1] Volker John[2] Christian Merdon[3]

We consider the finite element discretization of the Oseen equations. The LSVS convection stabilization proposed in [1], which is motivated by the underlying vorticity equation (obtained by applying the curl operator to the momentum equation) has the advantage that it leads for the Scott-Vogelius finite element to a pressure-robust method. We extended the LSVS method to the classical Taylor-Hood finite element space which is not pressure-robust. We added the grad-div stabilization to improve the mass conservation. The theoretical result in [2] stating that for large grad-div parameter γ the Taylor-Hood method converges to the Scott–Vogelius method is carried over to the LSVS grad-div stabilized scheme. In addition, by utilizing the already proved $O(h^{k+\frac{1}{2}})$ Scott-Vogelius error estimate from [1], we proved an error estimate for the velocity of the same order for Taylor-Hood finite elements. Numerical studies are performed to test the method and investigate the optimal choice of the LSVS and the grad-div stabilization parameters.

References:

[1] https://epubs.siam.org/doi/10.1137/20M1351230

[2] https://epubs.siam.org/doi/10.1137/100794250

[1]Freie Universität Berlin / Berlin Mathematical School
marwazinabdeen@gmail.com

[2]Freie Universität Berlin / Weierstrass Institute for Applied Analysis and Stochastics, Numerical Mathematics and Scientific Computing
john@wias-berlin.de

[3]Weierstrass Institute for Applied Analysis and Stochastics, Numerical Mathematics and Scientific Computing
Christian.Merdon@wias-berlin.de

Notes on session “CFD/DG”

Good Parallel in Time Methods for Hyperbolic Problems

Martin J. Gander[1]

Parallel in Time (PinT) methods have received a lot of attention over the past years, and there is an annual conference now dedicated to these techniques. Parallelizing a large scale computation in the time direction appears to be rather unusual at first sight, since time dependent problems obey a causality principle: the solution later in time is depending on the solution earlier in time, and never the other way round, so there does not seem to by any natural parallelism. Nevertheless, when parallelization in space for such problems saturates, parallelization in time appears tempting.

I will first show in my talk why for parabolic problems, parallelization in time is rather natural. This explains why there are in the meantime many successful PinT algorithms for such problems, like Parareal, Parareal-Schwarz-Waveform-Relaxation and Space-Time-Multigrid. In contrast, for hyperbolic problems, parallelization in time is much more challenging. This is because solutions of hyperbolic problems depend on their history in a much more stringent and detailed way than for parabolic problems that forget details over time. Nevertheless, several successful PinT algorithms have been developed for hyperbolic problems over the past decade, and I will explain three of them in my lecture: ParaExp, ParaDiag, and (Un)mapped Tent Pitching.

[1]Université de Genève
Martin.Gander@unige.ch

Notes on session “Parallel in Time”

Choosing starting vectors for Newton's method in nonlinear elasticity

Lina Fesefeldt[1] Sabine Le Borne[2]

In nonlinear elasticity, Newton's method is used to find a deformation that satisfies the equilibrium of external and internal forces. The convergence of Newton's method highly depends on the choice of the initial guess of the solution. In case of divergence, load steps can be used to stabilize the method: The force acting on the body is applied in increments, and each solution to the sub-problems serves as a new starting vector for the next load step. While this method is intuitive and established, it is also costly in computation time and memory. When the p-Version of FEM is used in combination with a hierarchical basis for the shape functions, a deformation always contains solutions for a lower maximum polynomial degree. We analyse the effect of using lower polynomial degrees on intermediate load steps on the convergence behaviour and overall computational costs.

[1]Hamburg University of Technology / Institute of Mathematics
lina.fesefeldt@tuhh.de

[2]Hamburg University of Technology / Institute of Mathematics
leborne@tuhh.de

Multigrid in H(curl) on Hybrid Tetrahedral Grids

Daniel Bauer[1]

This work presents theory and algorithms relevant to the solution of Maxwell's equations as well as their implementation in the massively scalable finite element framework *HyTeG*. We focus on multigrid methods for the curl-curl-problem which arises from the time-harmonic formulation of Maxwell's equations:

$$\alpha \, \mathbf{curl}\, \mathbf{curl}\, \boldsymbol{u} + \beta \boldsymbol{u} = \boldsymbol{f} \, , \tag{1}$$

where $\alpha, \beta > 0$. This problem is not elliptic, rendering standard multigrid smoothers ineffective.

We resort to finite element exterior calculus (FEEC) to explain our choice of discretization: linear Nédélec edge elements of the first kind. Furthermore, FEEC directly leads to the Hodge decomposition, which is pivotal for the design of effective multigrid smoothers in $\boldsymbol{H}(\mathbf{curl})$. These were pioneered by Hiptmair, who proposed smoothing both in the Nédélec finite element space and the space of scalar potentials approximated by piecewise Lagrangian polynomials.

Novel is our implementation of the Nédélec space and associated grid transfer operators from/to the space of piecewise Lagrangian polynomials in *HyTeG*. The code makes use of code generation techniques to go from a mathematical description of operators and bilinear forms to efficient compute kernels automatically. This enables us to perform certain optimizations like common subexpression elimination on the symbolic level.

HyTeG is a finite element framework designed for massively parallel compute architectures. It supersedes the HHG framework which was already capable of solving systems with 10^{13} unknowns. The key building block to achieve these impressive results is a matrix-free implementation of geometric multigrid on hybrid tetrahedral grids. Using successive uniform refinement of a coarse mesh, we obtain a hierarchy of nested grids for multigrid and fast, indirection-free code. At the same time, the flexibility of unstructured topology is recovered by transforming the grid on all refinement levels to the actual domain yielding a curvilinear mesh.

References:

[1] N. Kohl, D. Thönnes, D. Drzisga, D. Bartuschat, and U. Rüde, "The HyTeG finite-element software framework for scalable multigrid solvers," International Journal of Parallel, Emergent and Distributed Systems, vol. 34, no. 5, pp. 477–496, 2019, doi: 10.1080/17445760.2018.1506453.

[2] R. Hiptmair, "Multigrid Method for Maxwell's Equations," SIAM J. Numer. Anal., vol. 36, no. 1, pp. 204–225, Jan. 1998, doi: 10.1137/S0036142997326203.

[3] D. N. Arnold, Finite Element Exterior Calculus. Philadelphia, PA: Society for Industrial and Applied Mathematics, 2018. doi: 10.1137/1.9781611975543.

[1]Friedrich-Alexander-Universität Erlangen-Nürnberg, Lehrstuhl für Informatik 10 (Systemsimulation), Germany
daniel.j.bauer@fau.de

Matrix-free Implementation and Evaluation of the Enriched Galerkin Finite Element Method for the Stokes Problem with Varying Viscosity

Fabian Böhm[1]

The simulation of earth mantle convection with the Finite Element method (FEM) is a challenging task for computational engineers. In the discretization of the underlying mathematical model, the Stokes equation, large numbers of degrees of freedom (DoFs) are involved, which makes matrix-free codes necessary. Furthermore, strong variations and even discontinuities in the viscosity cause the linear system to be highly ill-conditioned, and FEM tend to produce unphysical oscillations. It is unclear, which particular FEM is the single best suited one to use for such problems. Higher order methods offer better approximation properties, like a faster convergence rate but also have a computationally more expensive operator application due to more DoFs and non-zeros. Lower order methods are cheaper in that regard but converge slower. Another criterium is inf-sup stability, which some candidates lack. The classical Taylor-Hood method (P2-P1) is a relevant candidate. A second candidate is the enriched Galerkin method (EG-P0), which uses linear polynomials and a vectorial enrichment for the velocity. It has the fewest DoFs per element of all mixed FEM while being inf-sup stable. In this work, EG-P0 is implemented for the first time with a matrix-free operator application. Additionally, the discrete operator is analyzed regarding the block structure, number of non-zeros per row, and the impact of the enrichment. Furthermore, the computational work for assembling the local matrix and stencil application is quantified. EG-P0 is compared with P2-P1 for analytical 2D and 3D test cases with smoothly varying viscosity, the so called SolVi and MultiSinker test cases. The criteria are how much computational work in flops must be invested to reach a certain error, the convergence rate of the discretization and the solution quality.

[1]Chair for Systemsimulation, FAU
Fabian.boehm@fau.de

Pairing Raviart-Thomas elements with conforming nodal elments in mixed finite element dicretizations

Goulm Pierre-Alain[1] [2] [3] [4]

The mixed finite element method has been widely utilized for solving elasticity problems, encompassing the development of several element pairs and various formulation types. In particular, mixed finite elements were introduce to allow for the robust implementation of constraints. Traditionally, an inf-sup condition is required to guarantee compatibility between finite element spaces. However, the recent contribution [1] shows existence and uniqueness of the solution can be obtained even though the numerical schemes is inf-sup unstable. This poster aims to investigate the stability of the corresponding elasticity problem by closely examining the inf-sup condition for the continuous element pair $(RT_0)^2 \times (P_1)^2$ introduced in [2]. By analyzing this specific combination of elements, we can shed light on the stability aspects of the problem at hand.

References:

[1] Daniele Boffi and Fleurianne Bertrand. A counterexample for the inf-sup stability of the RT^0/P^1 in $L_2H_1^0$ finite element combination for the mixed Poisson equation. Cambridge University Press, 2019. https://onlinelibrary.wiley.com/doi/abs/10.1002/pamm.201900426

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[1]TU Chemnitz
goulm@insa-toulouse.fr

[2]TU Chemnitz
goulm@insa-toulouse.fr

[3]TU Chemnitz
fleurianne.bertrand@mathematik.tu-chemnitz.de

[4]KAUST Computer, Electrical and Mathematical Sciences and Engineering Division
daniele.boffi@kaust.edu.sa

The scalar potential approach in nonlinear magnetostatics

Felix Engertsberger[1] Herbert Egger[2]

This talk covers the analysis of the scalar potential approach in the non-linear magnetostatic setting. First, the physical model is presented and the potential approach is motivated. In this master thesis, a very general form of the nonlinear constitutive equation is considered, which simplifies the analysis dramatically. Next, we derive the variational problem and state equivalent minimization and Lagrange multiplier problems. Existence & uniqueness is shown and a stability estimate is given. For the discretization standard courant elements are considered and the usage of numerical integration is justified with a nonlinear variation of the Strang lemma. Finally, first numerical results of an iron circuit in 2D are presented.

[1]NUMA, JKU, Altenbergerstraße 69, 4040 Linz
felix.engertsberger@jku.at

[2]RICAM, Altenbergerstraße 69, 4040 Linz
herbert.egger@ricam.oeaw.ac.at

Notes on session “Poster Pitches”

Total Generalized Variation with Finite Elements and Applications

Roland Herzog[1]

The total variation (TV) semi-norm is popular as a regularizing functional in inverse problems and imaging, favoring piecewise constant functions with few jumps. As an extension, Bredies, Kunisch and Pock introduced the total generalized variation (TGV) which favors piecewise linear (or higher-order polynomials). In this presentation, we address the discretization of second-order TGV with appropriate families of finite element functions. Moreover, we discuss algorithms for the numerical solution of associated imaging problems and show numerical results.

[1]Heidelberg University
roland.herzog@iwr.uni-heidelberg.de

Matrix-free Discontinuous Galerkin Solvers for the Cardiovascular System

Richard Schussnig[1] Martin Kronbichler[2]

The coupling of incompressible fluid and solid phases remains a challenging application especially in the medical context. Such problems frequently arise in medical device design, surgery planning or clinical support, but similar difficulties are encountered in more general settings as well, when the density of the involved fluid and structure are similar. Standard fluid-structure interaction algorithms suffer from a high added-mass effect in addition to the large number of spatial and temporal unknowns to accurately represent the physical processes. Hence, robust and efficient numerical tools reducing the computational burden and the required time to solution are urgently needed.

Adopting monolithic solvers, which are traditionally applied in this field, preconditioning the linear system presents major difficulties. Following the partitioned approach, on the other side, a strong added-mass effect leads to increased iteration counts in the coupling algorithm. However, recently developed methods [1] combining Robin coupling conditions, interface quasi-Newton methods and semi-implicit coupling strategies offer an attractive alternative to monolithic schemes. Within this contribution, we present recent developments combining matrix-free Discontinuous Galerkin solvers within the open-source software framework ExaDG [2] with accelerated partitioned schemes in practical applications to patient-specific cardiovascular models. Higher-order discretizations for the three-dimensional structure and fluid are considered, comparing various alternative formulations of the fluid subproblem leading to monolithic velocity-pressure systems or, as an alternative, pressure-correction schemes splitting the problems governing fluid velocity and pressure, see, e.g., Kronbichler et al. [3].

We present results demonstrating the robustness of the solution with respect to the Robin parameter and investigate its interplay with the semi-implicit variants of the strongly coupled and accelerated partitioned solver. Practical relevance is demonstrated in a clinical scenario of blood flow through an iliac bifurcation, including relevant modeling aspects such as physiological boundary conditions and realistic problem parameters.

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[1]University of Augsburg, Institute of Mathematics
richard.schussnig@uni-a.de

[2]University of Augsburg, Institute of Mathematics
martin.kronbichler@uni-a.de

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A finite volume method for transport induced neurite growth

Max Winkler[1] Greta Marino[2] Jan-Frederick Pietschmann[3]

In this talk we study a free boundary model for vesicle transport on neurites taking into account neurite growth and shrinkage as well. The model consists of two PDEs describing bidirectional transport of retrograde and anterograde vesicles, ODEs describing the concentrations in the soma and the growth cones at the end of the neurites, as well as an ODE encoding a growth and shrinkage of the neurite. We give some existence and uniqueness results for the equation system, discuss steady state solutions, present a numerical computation scheme based on a finite volume discretization and compare the simulation results with biological experiments.

[1]TU Chemnitz
max.winkler@mathematik.tu-chemnitz.de

[2]Universität Augsburg
greta.marino@mathematik.tu-chemnitz.de

[3]Universität Augsburg
jan-f.pietschmann@uni-a.de

On discrete ground states of rotating Bose–Einstein condensates

Mahima Yadav[1] Patrick Henning[2]

The talk focuses on the study of ground states of Bose–Einstein condensates in a rotating frame. The ground states are described as the constrained minimizers of the Gross-Pitaevskii energy functional with an angular momentum term. The problem is discretized using Lagrange finite element spaces of arbitrary polynomial order. The approximation properties of discrete ground states are presented, taking into account the missing uniqueness of ground states which is mainly caused by the invariance of the energy functional under complex phase shifts. The error analysis is based on an Euler–Lagrange functional that we restrict to certain tangent spaces in which we have local uniqueness of ground states. Error estimates of optimal order are shown for the L^2- and H^1-norm, as well as for the ground state energy and chemical potential. We also present numerical experiments to illustrate various aspects of the problem structure.

[1]Ruhr-University Bochum, Department of Mathematics
mahima.yadav@rub.de

[2]Ruhr-University Bochum
patrick.henning@rub.de

Notes on session “Applications & Optimal Control”

A Space-Time Fast Boundary Element Method for the Heat Equation with Temporal Nearfield Compression

Günther Of[1] Raphael Watschinger[2]

We consider a space-time boundary element method for the solution of initial boundary value problems of the heat equation in three spatial dimensions. In particular we deal with tensor product meshes with adaptive decompositions of the considered time interval and adaptive spatial meshes. We apply a space-time fast multipole method as well as shared and distributed memory parallelization with respect to space and time.

We present a novel temporal nearfield compression technique which enables efficient computations for fine spatial mesh resolutions related to the considered adaptive tensor product meshes. In particular, we introduce a version of the adaptive cross approximation tailored to the nature of the considered heat kernel. Finally, we present numerical experiments that demonstrate the great benefits of the new method for tensor product meshes with spatially fine meshes and adaptive spatial meshes.

[1]Graz University of Technology, Institute of Applied Mathematics, Austria
of@tugraz.at

[2]Graz University of Technology, Institute of Applied Mathematics, Austria
watschinger@math.tugraz.at

Space Time Dual Weighted Residual Error Estimation

Bernhard Endtmayer[1] Ulrich Langer[2] Andreas Schafelner[3]

In this talk, we derive goal oriented error estimation based on the dual weighted residual method for Space-time problems. For this error estimator, we require either a soltion in an enriched space or an interpolation of the solution. Under a saturation assumption based on the enriched space or the interpolation the resulting error estimator is efficient and reliable. Finally, we will conclude the talk with numerical results, featuring a time-dependent p-Laplace equation in 2D and 3D.

[1]IfAM, AG Numerische Mathematik , Leibnitz University Hannover
endtmayer@ifam.uni-hannover.de

[2]NUMA, Institut für Numerische Mathematik, Johannes Kepler University Linz
ulanger@numa.uni-linz.ac.at

[3]NUMA, Institut für Numerische Mathematik, Johannes Kepler University Linz
andreas.schafelner@jku.at

Unique continuation for the wave equation using a discontinuous Galerkin time discretization

Janosch Preuss[1] Erik Burman[2]

We revisit a unique continuation problem for the wave equation in the time domain which has previously been considered in [1]. In the latter publication this problem has been solved using a full space time discretization. For computational efficiency it would be highly desirable if the time discretization could instead be realized by a discontinuous Galerkin method. This is known to allow for time-marching procedures provided only upwind-type couplings are present in the time discretization. Unfortunately, our investigations show that some stronger couplings in time appear to be necessary to preserve the optimal error estimates shown in [1]. However, by identifying which of those couplings are essential and dropping the others we managed to relax the scheme to an extent where time-marching is at least applicable as a preconditioner. The performance of this preconditioner, which may be interpreted as a forward sweep in the time domain, is illustrated in numerical experiments.

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[1]University College London, Department of Mathematics
j.preuss@ucl.ac.uk

[2]University College London, Department of Mathematics
e.burman@ucl.ac.uk

Post-processing and improved error estimates of numerical methods for evolutionary systems

Sebastian Franz[1]

We consider evolutionary systems, i.e. systems of linear partial differential equations arising from the mathematical physics, in the form

$$(\partial M_0 + M_1 + A)U = F$$

where M_0, M_1 are bounded linear self-adjoint operators on a Hilbert space $H = H(\Omega)$ and A is a skew self-adjoint operator on H. Suppose further that there are constants ρ_0 and $\gamma > 0$, such that

$$\rho M_0 + M_1 \geq \gamma$$

for all $\rho \geq \rho_0$. For these systems there exists a general solution theory, see [1, Solution Theory] in exponentially weighted spaces which can be exploited in the analysis of numerical methods.

The numerical method considered is a discontinuous Galerkin method in time combined with a conforming Galerkin method in space. Building on our recent paper [2], we improve some of the results, study the dependence of the numerical solution on the weight-parameter, and consider a reformulation and post-processing of its numerical solution. Numerical simulations support the theoretical findings.

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[1]TU Dresden, Institute of Scientific Computing
sebastian.franz@tu-dresden.de

Notes on session “Space-Time”

Optimization based formulation and solution of inverse problems

Barbara Kaltenbacher[1]

The probably most well-known and most widely used approach to solving inverse problems is by combined minimization of data misfit and some regularization term, usually referred to as Tikhonov-Philips regularization. Still, this relies on the use of some forward operator, which is the concatenation of the observation operator with the parameter-to-state-map for the underlying model. Recently, all-at-once formulations have been considered as an alternative to this reduced formulation, avoiding the use of a parameter-to-state map, which would sometimes lead to too restrictive conditions. Here the model and the observation are considered simultaneously as one large system with the state and the parameter as unknowns. A still more general formulation of inverse problems, containing both the reduced and the all-at-once formulation, but also the well-known and highly versatile so-called variational approach (not to be mistaken with variational regularization) as special cases, is to formulate the inverse problem as a minimization problem (instead of an equation) for the state and parameter. Regularization can be incorporated via imposing constraints and/or adding regularization terms to the objective. In this talk, after providing the general setting with convergence results, we will discuss some examples and in particular dwell on some applications in (nonlinear) acoustics.

[1]University of Klagenfurt, Department of Mathematics
barbara.kaltenbacher@aau.at

Notes on session “Inverse Problems”

Benchmarking Hybrid Finite Element/Deep Neural Networks and Classical Finite Element Methods

Nils Margenberg[1] Robert Jendersie[2] Christian Lessig[3] Thomas Richter[4]

Accurate flow simulations remain a challenging task. Combining classical finite element approximation techniques with deep neural neworks adds new aspects to the pure numerics-oriented approach and offers potential for further innovations. In this talk we discuss the use of deep neural networks for augmenting classical finite element simulations in fluid-dynamics.

We first establish new benchmark results for the classical DFG-benchmark in 3D using classical finite element simulations with high accuracy. We extend these settings to higher Reynolds numbers and compare two different FEM libraries: Gascoigne3D and deal.II. We compare the computation of drag and lift forces across the two software platforms and show that they are in good agreement.

At high Reynolds numbers, accurate simulations in 3D settings become increasingly difficult, and the classical methods reach their limits. To address this issue, we discuss approaches to connect the finite element method with neural networks. We propose the Deep Neural Network Multigrid Solver, which combines a geometric multigrid solver with a deep neural network to overcome limitations of classical methods. This approach uses classical simulation techniques where their strengths are eminent, such as the efficient representation of a coarse, large-scale flow field. Neural networks are used when a full resolution of the effects does not seem possible or efficient.

We demonstrate the efficiency, generalizability, and scalability of our proposed approach using 3D simulations. Our focus is particularly on issues of stability, generalizability, and error accuracy, and we establish the error accuracy of our proposed method by comparing it with the newly established benchmark results. Overall, our approach offers potential for further innovations in accurate flow simulations.

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[1]Helmut Schmidt Universität, Numerische Mathematik
margenbn@hsu-hh.de

[2]University of Magdeburg, Institute for Simulation and Graphics
robert.jendersie@ovgu.de

[3]University of Magdeburg, Institute for Simulation and Graphics
christian.lessig@ovgu.de

[4]University of Magdeburg, Institute for Analysis and Numerics
thomas.richter@ovgu.de

Deep learning approaches based on HDG method for solving some nonlinear elliptic equations

Reza Mokhtari[1] Shima Baharlouei[2]

This talk aims to introduce and analyze two deep neural networks (DNNs) approaches based on the hybridized discontinuous Galerkin (HDG) method for solving some nonlinear elliptic equations. Many known robust numerical methods such as different types of HDG methods have a high dependency on mesh- grid points that makes serious difficulties in problems with complex geometry, especially in high dimensions. Recently, we have constructed two approaches that use artificial neural network approaches to overcome this defect of the classical methods, especially HDG methods. In the first approach, which we called DNN-HDG, after applying the HDG method with a suitable definition of numerical flux and trace, the variational form solutions are approximated directly using the neural networks idea. The second approach, known as DNN-HDG-II, is more compatible with the classical HDG method, in the sense that solutions are considered as linear combinations of the trial functions and then coefficients are approximated using the neural network technique. In this talk, we intend to extend these two efficient and robust methods for solving the following nonlinear elliptic problem

$$\begin{aligned} -\nabla \cdot (\kappa(\mathfrak{u}, \boldsymbol{x})\, \nabla \mathfrak{u}(\boldsymbol{x})) &= \mathfrak{f}(\mathfrak{u}, \boldsymbol{x}), && \boldsymbol{x} \in \Omega \subset \mathbb{R}^d, \\ \mathfrak{u}(\boldsymbol{x}) &= \mathfrak{g}_{_D}(\boldsymbol{x}), && \boldsymbol{x} \in \partial\Omega_D, \\ -\kappa(\mathfrak{u}, \boldsymbol{x})\, \nabla \mathfrak{u}(\boldsymbol{x}) \cdot \boldsymbol{n} &= \mathfrak{g}_{_N}(\boldsymbol{x}), && \boldsymbol{x} \in \partial\Omega_N, \end{aligned} \tag{1}$$

where $d \in \mathbb{N}$ is the spatial dimension, $\boldsymbol{x} = (x_1, \ldots, x_d)^T$, $\boldsymbol{n}$ is the outward unit normal vector, and $\partial\Omega_D$ and $\partial\Omega_N$ are parts of the boundary with Dirichlet and Neumann boundary conditions, respectively. Also, functions κ and $\mathfrak{f}$ are nonlinear terms that are assumed to be in suitable function spaces. We prove that the loss function corresponding to the proposed DNN-HDG methods for solving (1) converges to zero as the mesh step size reduces. Moreover, through some examples, we show that the DNN-HDG methods can efficiently and accurately extract the pattern of the solutions in one, two, and three dimensions. Also, some precious advantages of the DNN-HDG methods compared to the classical HDG methods will be demonstrated, especially for problems with noisy data. Likewise, we demonstrate the ability of the DNN-HDG methods in problems whose exact solutions are not accessible.

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[1]Department of Mathematical Sciences, Isfahan University of Technology, Isfahan 84156-83111, Iran
mokhtari@iut.ac.ir

[2]Department of Mathematical Sciences, Isfahan University of Technology, Isfahan 84156-83111, Iran
s.baharloui@math.iut.ac.ir

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Identifying cracks in membranes via their eigenfrequencies - A theoretical and practical approach

Philipp Zilk[1] Thomas Apel[2]

The eigenfrequencies of a vibrating membrane generally depend on its shape. The associated inverse problem which became famous through the work of Kac has been widely discussed in the literature. We look at this problem in the context of cracks. Is it possible to identify a crack in a membrane when its eigenfrequencies are known?

First, we show a theoretical approach to see that it is indeed possible to do so under certain conditions. Then, we present a practical method to confirm our theoretical findings. Concretely, we train a neural network using simulated data to predict the shape of a crack from the corresponding eigenfrequencies. For the simulations we use Isogeometric Analysis, which is well known for its excellent spectral approximation properties. Some of the resulting eigenfunctions have a singularity of type r^{ν}, thus the corresponding eigenvalues can not be approximated well with uniform refinement procedures. Therefore, we introduce a mesh grading approach based on a singular isogeometric mapping and illustrate optimal convergence order for the eigenfunctions and eigenvalues.

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[1]University of the Bundeswehr Munich, Institute for Mathematics and Computer-Based Simulation
philipp.zilk@unibw.de

[2]University of the Bundeswehr Munich, Institute for Mathematics and Computer-Based Simulation
thomas.apel@unibw.de

Non-coercive boundary value problems

Thomas Apel[1] Mariano Mateos[2] Arnd Rösch[3]

Standard tools like the Lax–Milgram lemma or the Céa lemma are based on the coercivity of the bilinear form. A typical assumption for a scalar partial differential equation of second order is $c - \frac{1}{2}\operatorname{div} b > 0$ to ensure coercivity. However, this assumption can be too restrictive when $\operatorname{div} b$ is large.

In this contribution the solution and its finite element approximation are examined for the Neumann boundary value problem in such a case: existence and regularity of the solution in weighted Sobolev spaces, discretization with graded meshes, error estimates in the domain and on the boundary, numerical tests.

The application of these insights to a Neumann optimal control problem is discussed as well. Note that for the analysis of optimal control problems the adjoint problem is used, and the problem under consideration is not self-adjoint.

[1]Universität der Bundeswehr München, Institute for Mathematics and Computer-Based Simulation (IMCS)
thomas.apel@unibw.de

[2]Universidad de Oviedo, Departamento de Matemáticas, Gijón, Spain
mmateos@uniovi.es

[3]Universität Duisburg-Essen, Fakultät für Mathematik, Essen, Germany
arnd.roesch@uni-due.de

Mass-lumping discretization and solvers for distributed elliptic optimal control problems with L_2 - regularization

Ulrich Langer[1] Richard Löscher[2] Olaf Steinbach[3] Huidong Yang[4]

The purpose of this talk is to investigate the effects of the use of mass-lumping in the finite element discretization of the reduced first-order optimality system arising from a standard tracking-type, distributed elliptic optimal control problem with L_2 regularization. We show that mass-lumping will not affect the L_2 error between the desired state and the computed state, but will lead to a Schur-complement system that allows for a fast matrix-by-vector multiplication. We show that the use of the Schur-Complement Preconditioned Conjugate Gradient method in a nested iteration setting leads to an asymptotically optimal solver with respect to the complexity. Moreover, it is easy to parallelize this solver.

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[1]Johannes Kepler University Linz, Institute of Numerical Mathematics
ulanger@numa.uni-linz.ac.at

[2]Technische Universität Graz, Institut für Angewandte Mathematik,
loescher@math.tugraz.at

[3]Technische Universität Graz, Institut für Angewandte Mathematik,
o.steinbach@tugraz.at

[4]University of Vienna, Faculty of Mathematics, and Doppler Laboratory for Mathematical Modeling and Simulation of Next Generations of Ultrasound Devices
huidong.yang@univie.ac.at

Notes on session “Machine Learning / Optimal Control”

Nonlinear FETI-DP and Quasi-Newton Methods

Stephan Köhler[1] Oliver Rheinbach[2]

Nonlinear FETI-DP (Finite Element Tearing and Interconnecting – Dual-Primal) methods are domain decomposition methods for the solution of nonlinear finite element problems. In the classical Newton-Krylov-Domain-Decomposition approach, the problem of interest is linearized first and then a domain decomposition method, such as Balancing Domain Decomposition (BDD), Balancing Domain Decomposition by Constraints (BDDC), Finite Element Tearing and Interconnecting (FETI-1), FETI–DP or overlapping Schwarz methods, is used for the solution of the Newton system. In nonlinear domain decomposition method, this order is interchanged. In a first step a nonlinear domain decomposition is built, this provides nonlinear subproblems and, possibly, a nonlinear coarse problem. Afterwards the nonlinear decomposition is linearized.

Quasi-Newton methods compute a solution of the discretized problem by using quadratic subproblems. These subproblems are obtained by updating some initial approximation of the Hessian with the gradient of the objective. This provides superlinear convergence and, by the Sherman-Morrison-Woodbury formula, a fast computation of the quasi-Newton update for the current iterate.,

This talk discusses the combination of Nonlinear FETI-DP and quasi-Newton methods.

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[1]Technische Universität Bergakademie Freiberg, Institut für Numerische Mathematik und Optimierung
stephan.koehler@math.tu-freiberg.de

[2]Technische Universität Bergakademie Freiberg, Institut für Numerische Mathematik und Optimierung
oliver.rheinbach@math.tu-freiberg.de

Isogeometric multilevel quadrature for forward and inverse random acoustic scattering

Helmut Harbrecht[1] Jürgen Dölz[2] Carlos Jerez-Hanckes[3] Michael Multerer[4]

This talk is concerned with the numerical solution of forward and inverse acoustic scattering problems by randomly shaped obstacles in three-dimensional space using a fast isogeometric boundary element method. Within the isogeometric framework, realizations of the random scatterer can efficiently be computed by simply updating the NURBS mappings which represent the scatterer. This way, we end up with a random deformation field. In particular, we show that the knowledge of the deformation field's expectation and covariance at the surface of the scatterer are already sufficient to compute the surface Karhunen-Loève expansion. Leveraging on the isogeometric framework, we utilize multilevel quadrature methods for the efficient approximation of quantities of interest, such as the scattered wave's expectation and variance. Computing the wave's Cauchy data at an artificial, fixed interface enclosing the random obstacle, we can also directly infer quantities of interest in free space. Adopting the Bayesian paradigm, we finally compute the expected shape and the variance of the scatterer from noisy measurements of the scattered wave at the artificial interface. Numerical results for the forward and inverse problem are given to demonstrate the feasibility of the proposed approach.

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[1]University of Basel, Department of Mathematics and Computer Science
helmut.harbrecht@unibas.ch

[2]Institute for Numerical Simulation, Universty of Bonn
doelz@ins.uni-bonn.de

[3]Faculty of Engineering and Sciences, Universidad Adolfo Ibáñez
carlos.jerez@uai.cl

[4]Faculty of Informatics, USI Lugano
michael.multerer@usi.ch

Solving Elliptic Partial Differential Equations on Metric Graphs using Multigrid Methods

Max Brockmann[1]

The question of investigating partial differential equations (PDEs) on graphs arises in the context of an interdisciplinary research project of the prediction of protein propagation in the brain network of Alzheimer's Disease patients [3]. Graphs allow modelling the structure of the brain network, while PDEs, particularly diffusion equations, describe the protein propagation. As a first attempt to approximate such PDEs on graphs, we focus on second order elliptic PDEs and multigrid methods for the solution of systems arising from a finite element discretization.
In order to formulate PDEs on graphs, we explain the network structure with the help of metric graphs. Metric graphs use an edgewise parameterization of the graph such that differential operators can be defined on graphs. Additionally, we require Neumann-Kirchhoff Conditions, a kind of flow conservation property, on all vertices.
We discretize the metric graph with a finite element method, as described in [1]. The discretization of the metric graph can be interpreted as an extended graph with additional vertices. We then choose a hat function basis on the extended graph, resulting in a characterisation of the weak formulation of the PDE as as

$$\begin{pmatrix} \mathbf{H}_{\mathcal{E}\mathcal{E}} & \mathbf{H}_{\mathcal{E}\mathcal{V}} \\ \mathbf{H}_{\mathcal{V}\mathcal{E}} & \mathbf{H}_{\mathcal{V}\mathcal{V}} \end{pmatrix} \mathbf{u} = \mathbf{f},$$

where $\mathbf{u}$ is the coefficient vector of the solution of the PDE written in its basis.
Each submatrix $\mathbf{H}_{\mathcal{E}\mathcal{E}}, \mathbf{H}_{\mathcal{E}\mathcal{V}}$, and $\mathbf{H}_{\mathcal{V}\mathcal{V}}$ corresponds to different adjacency of hat functions on the extended graph. Thus their size increase with more discretization points on each of the edges. This is especially important for the matrix $\mathbf{H}_{\mathcal{E}\mathcal{E}}$, because it is a block- diagonal matrix, with each blocks size proportional to the number of discretization points. Consequently, a fine discretization leads to a large system of equations and high computational cost.
We use a multigrid method to find the solution of the system of equations, making necessary adjustments for intergrid operators on graphs.
We show numerical results of the convergence rate of the multigrid method on Test problems.

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[1]University of Cologne, Department of Mathematics and Computer Science, Division of Mathematics
brockmann@math.uni-koeln.de

Finite Element Simulation for Elastic and Plastic Fluids

Muhammad Tayyab Bin Saghir[1] Hogenrich Damanik[2] Stefan Turek[3]

In this study, we present the development of a 2D finite-element solver for the simulation of fluids exhibiting both elastic and plastic constitutive properties. These types of fluids are commonly modeled in the constitutive communities as elastoviscoplastic fluids, where the numerical variables depend on the choice of various constitutive models. Many of these constitutive models involve different numerical frameworks to handle the transition between the solid and liquid phases, such as the Saramito model. Although these strategies offer improved accuracy, the numerical treatment becomes significantly more complex, primarily due to the challenges of identifying the interface between the two phases.

To address these complexities, we propose a similar approach from Emad Chaparian et al. by combining the constitutive models of the Oldroyd-B model and the Papanastasiou model for Bingham fluids within a single Eulerian numerical framework. This strategy, which has recently gained attention in elastoviscoplasticity studies, has demonstrated promising qualitative results. Within this approach, our aim is to approximate the velocity, pressure, and elastic stresses in both space and time. To achieve this, we employ a high-order finite element method for the velocity-stress approximation and a discontinuous pressure element. This specific element pair has proven to be highly effective for accurately capturing the behavior of both the Oldroyd-B and Bingham fluids, including nonlinear viscosity functions.

Our study consists of two main steps. First, we validate each component of the numerical solver individually, ensuring that the approximations and calculations are accurate. This step is crucial to establish the reliability and robustness of our approach. Subsequently, in the second step, we apply the solver to simulate elastoviscoplastic fluid behavior in a porous medium. By investigating the fluid flow and deformation within this specific context, we aim to demonstrate the capabilities and potential of our methodology.

[1]Institute of Applied Mathematics (LS III), TU Dortmund University, Vogelpothsweg 87, D-44227 Dortmund, Germany.
msaghir@math.tu-dortmund.de

[2]Institute of Applied Mathematics (LS III), TU Dortmund University, Vogelpothsweg 87, D-44227 Dortmund, Germany.
hdamanik@math.uni-dortmund.de

[3]Institute of Applied Mathematics (LS III), TU Dortmund University, Vogelpothsweg 87, D-44227 Dortmund, Germany.
stefan.turek@mathematik.tu-dortmund.de

Finite Element Methods for Curvature Computation

Joachim Schöberl[1]

In this talk we present recent results on discretizing curvature by finite element methods. One application is the simulation of elastic shells, where the bending energy is described by the change of curvature with respect to a reference configuration. Here we need extrinsic curvature of embedded manifolds, i.e. of surface meshes in R3. In the second part we talk about intrinsic curvature, where only the metric information inside the surface is available. Intrinsic curvature is needed also for numerical relativity. Both approaches need the concept of distributional derivatives. We show that finite elements of Hellan-Herrmann-Johnson and Regge-type match with these distributions. We show how to use these methods within the NGSolve finite element package.

[1]Technical University of Vienna
joachim.schoeberl@tuwien.ac.at

Notes on session “Solvers / FEM-BEM”

Algebraic multigrid for finite element discretizations

Yvan Notay[1]

Algebraic multigrid (AMG) methods are known as efficient to solve linear systems resulting from low order finite element (FE) approximations of elliptic PDEs. However, just going to moderate order (say, P3, P4) may raise difficulties. On the other hand, non elliptic problems cannot be tackled directly by AMG methods.

In this lecture, we show how these difficulties may be met using aggregation-based AMG, considering moderate order FE on the one hand, and Stokes problems on the other hand. In both cases, the key is to combine theoretical analysis that guaranties to be "on the safe side" with efficient coding (including many heuristics) that allow one to obtain robust results on real life applications.

[1]Université Libre de Bruxelles, Service de Métrologie Nucléaire
yvan.notay@ulb.be

Notes on session “Algebraic Multigrid Methods”

Adaptive least-squares space-time finite element methods

Olaf Steinbach[1] Christian Köthe[2] Richard Löscher[3]

We consider the numerical solution of an operator equation $Bu = f$ by using a least-squares approach. We assume that $B : X \to Y^*$ is an isomorphism, and that $A : Y \to Y^*$ implies a norm in Y, where X and Y are Hilbert spaces. The minimizer of the least-squares functional $\frac{1}{2}\|Bu - f\|^2_{A^{-1}}$, i.e., the solution of the operator equation, is then characterized by the gradient equation with an elliptic and self-adjoint operator $S = B^*A^{-1}B : X \to X^*$. When introducing the adjoint $p = A^{-1}(f - Bu)$ we end up with a saddle point formulation to be solved numerically by using mixed finite element methods. Based on a discrete inf-sup stability condition we derive related a priori error estimates. While the adjoint p is zero by construction, its approximation p_h serves as a posteriori error indicator to drive an adaptive discretization scheme. While this approach can be applied to rather general equations, here we consider second order linear partial differential equations, including the Poisson equation, the heat equation, and the wave equation, in order to demonstrate the potential of this proposed approach which allows us to use almost arbitrary space-time finite element methods for the adaptive solution of time-dependent partial differential equations.

[1]TU Graz, Institute of Applied Mathematics
o.steinbach@tugraz.at

[2]TU Graz, Institute of Applied Mathematics
c.koethe@tugraz.at

[3]TU Graz, Institute of Applied Mathematics
loescher@math.tugraz.at

On the necessity of the inf-sup condition for a mixed finite element formulation

Fleurianne Bertrand[1] Daniele Boffi[2]

We study a non standard mixed formulation of the Poisson problem, sometimes known as dual mixed formulation. For reasons related to the equilibration of the flux, we use finite elements that are conforming in H(div) for the approximation of the gradients, even if the formulation would allow for discontinuous finite elements. The scheme is not uniformly inf-sup stable, but we can show existence and uniqueness of the solution, as well as optimal error estimates for the gradient variable when suitable regularity assumptions are made. Several additional remarks complete the paper, shedding some light on the sources of instability for mixed formulations.

References:

[1] https://arxiv.org/abs/2206.06968

[1]TU Chemnitz, Germany
fleurianne.bertrand@mathematik.tu-chemnitz.de

[2]Kaust, Saudi-Arabia
daniele.boffi@kaust.edu.sa

A pollution-free ultra-weak FOSLS discretization of the Helmholtz equation

Harald Monsuur[1] Rob Stevenson[2]

We consider an ultra-weak first order system discretization of the Helmholtz equation. By employing the optimal test norm, the 'ideal' method yields the best approximation to the pair of the Helmholtz solution and its scaled gradient w.r.t. the norm on $L_2(\Omega) \times L_2(\Omega)^d$ from the selected finite element trial space. On convex polygons, the 'practical', implementable method is shown to be pollution-free when the polynomial degree of the finite element test space grows proportionally with $\log \kappa$. Numerical results also on other domains show a much better accuracy than for the Galerkin method.

References:

[1] https://arxiv.org/abs/2303.16508

[1]Universiteit van Amsterdam
harald.monsuur@hotmail.com

[2]Universiteit van Amsterdam
r.p.stevenson@uva.nl

Derivation and simulation of thermoelastic Kirchhoff plates

Johanna Beier[1]

Within the research of the Cluster of Excellence PhoenixD it is of interest to simulate thermoelastic materials on thin optical components which have the structure of Kirchhoff-Plates. This leads to a bothsided nonlinear coupled 4th order system of the heat equation and the elasticity equations. The standard finite element method (FEM) is a powerful tool for the numerical solution of boundary value problems of elliptic PDEs. In this talk I will present a derivation of a 2nd order thermoelastic system on Kirchhoff-Plates following the method of Rafetseder and Zulehner. Further I will summarize some theoretical statements and show our FEM simulation results.

References:

[1] https://doi.org/10.1137/17M1118427

[1]Leibniz Universität Hannover, Institut für Angewandte Mathematik
beier@ifam.uni-hannover.de

Notes on session “Numerical Analysis”

Physics-based block preconditioning for mixed dimensional beam/solid coupling

Max Firmbach[1] Alexander Popp[2] Matthias Mayr[3]

Multigrid methods are known to be highly efficient and scalable preconditioners, not only for single-field problems but also for mortar methods ranging from contact mechanics to meshtying problems. An interesting application in this field is the mixed-dimensional coupling of slender beam structures embedded into three-dimensional solid bodies. Such beam/solid interactions can be found in several engineering scenarios (e.g. fiber-reinforced composite materials or reinforced concrete). Imposing the coupling constraints via a penalty method as proposed in leads to an ill-conditioned and highly non-diagonal dominant matrix. Solving such problems in an efficient and scalable manner with iterative solvers is difficult and makes an appropriate preconditioning essential. Using algebraic multigrid methods (AMG) in this context is yet again challenging as there are several open questions regarding AMG for beams and the coarsening of the mixed-dimensional coupling terms.

This talk will discuss a physics-based block preconditioning approach based on AMG. The outer block iteration is taken care of with an inexact LU-decomposition. A crucial part of the calculation is the approximation of the inverse appearing in the Schur complement. Using a sparse approximate inverse approach based on an appropriate sparsity pattern helps to retain a low iteration number and parallel scalability. Multilevel ideas will be used to approximate the block inverses appearing in the system. We will assess the performance and the weak scalability of the proposed block preconditioner using examples of the interaction of solids with torsion-free Kirchhoff-Love and Simo-Reissner beam finite elements.

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[1] https://onlinelibrary.wiley.com/doi/10.1002/nme.6680?af=R

[2] https://link.springer.com/article/10.1007/s00466-020-01907-0

[3] https://epubs.siam.org/doi/abs/10.1137/S1064827594276552?journalCode=sjoce3

[1]University of the Bundeswehr, Civil Engineering and Environmental Sciences
max.firmbach@unibw.de

[2]University of the Bundeswehr, Civil Engineering and Environmental Sciences
alexander.popp@unibw.de

[3]University of the Bundeswehr, Civil Engineering and Environmental Sciences
matthias.mayr@unibw.de

Solving fractional Burgers equations using the Hopf-Cole transformation and local discontinuous Galerkin method

Mohadese Ramezani[1] Reza Mokhtari[2] Gundolf Haase[3]

We study the following time-fractional Burgers equation

$$\begin{aligned} &D_t^\alpha u + \partial_x\left(\frac{u^2}{2}\right) - \mu\Delta u = 0, \qquad (x,t) \in \Omega \times (0,T], \\ &u(\cdot,t)|_{\partial\Omega} = 0, \qquad t \in [0,T], \\ &u(x,0) = u_0(x), \qquad x \in \Omega, \end{aligned}$$

in which Ω is the spatial domain and $D_t^\alpha u$ is the time fractional derivative of order $\alpha \in (0,1)$, i.e.

$$D_t^\alpha u(\cdot,t) = \frac{1}{\Gamma(2-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial u(\cdot,s)}{\partial s}\, \mathrm{d}s.$$

Recently, Li et al. considered solving the time-fractional Burgers equation using the local discontinuous Galerkin (LDG) method in space and L1 approximation in time. Here, we aim to consider solving this problem with different approaches. By using the Hopf-Cole transformation, the original time-fractional burgers equation is transformed into a subdiffusion equation with the Neumann boundary conditions. Moreover, the solution of the subdiffusion equation sometimes has low-order regularity in time even with smooth initial data. Here, we are concerned with both problems whose solutions have strong regularity and weak singularity. Together with a local discontinuous Galerkin method and a finite difference method (FDM) on a uniform mesh in time for discretizing the spatial and temporal derivatives, respectively, we obtain the numerical solution. Handling the singularities in the typical solution of subdiffusion problems, we establish the FDM on a non-uniform mesh for approximating the Caputo derivative and utilize the LDG method in space direction to derive the fully discrete method. Both numerical schemes have convergence spatial rate $\mathcal{O}(h^{k+1})$ when piecewise polynomials of degree k are used. The Caputo approximations used here are higher order than the L1 formula utilized by Li and his coworkers.

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[1] Li, C., Li, D., and Wang, Z. (2021). L1/LDG method for the generalized time-fractional Burgers equation. Mathematics and Computers in Simulation, 187, 357-378

[2] Li, C., Li, D., and Wang, Z. (2022). L1/LDG method for the generalized time-fractional Burgers

[1]Department of Mathematical Sciences, Isfahan University of Technology, Isfahan 84156-83111, Iran
mohadeseh.ramezani@alumni.iut.ac.ir

[2]Department of Mathematical Sciences, Isfahan University of Technology, Isfahan 84156-83111, Iran
mokhtari@iut.ac.ir

[3]Institute for Mathematics and Scientific Computing, University of Graz, Graz, Austria
gundolf.haase@uni-graz.at

equation in two spatial dimensions. Communications on Applied Mathematics and Computation, 1-24.

Implicit-explicit time discretization for Oseen's equation at high Reynolds number with application to fractional step methods

Deepika Garg[1] Erik Burman[2] Johnny Guzman[3]

In this talk we consider the application of implicit-explicit (IMEX) time discretizations for the incompressible Oseen equations. The pressure velocity coupling and the viscous terms are treated implicitly, while the convection term is treated explicitly. Both the second order backward differentiation and the Crank-Nicolson methods are considered for time discretization, resulting in a scheme similar to Gear's method on the one hand and to Adams Bashforth of second order on the other. For the discretization in space we consider finite element methods with stabilization on the gradient jumps. The stabilizing terms ensures inf-sup stability for equal order interpolation and robustness at high Reynolds number. Under suitable Courant conditions we prove stability of Gear's scheme in this regime. The stabilization allows us to prove error estimates of order $O(h^{k+\frac{1}{2}} + \tau^2)$. Here h is the mesh parameter, k the polynomial order and τ the time step. Finally we show that for inviscid flow (or underresolved viscous flow) the IMEX scheme can be written as a fractional step method in which only a mass matrix is inverted for each velocity component and a Poisson type equation is solved for the pressure.

[1]University College London
d.garg@ucl.ac.uk

[2]University College London
e.burman@ucl.ac.uk

[3]Brown University
johnny_guzman@brown.edu

Least-Squares Finite Element Methode for a non-linear Sea-Ice problem

Henrik Schneider[1] Fleurianne Bertrand[2]

A nonlinear sea-ice problem is considered in a least-squares finite element setting. The corresponding variational formulation approximating simultaneously the stress tensor and the velocity is analysed. In particular, the least-squares functional is coercive and continuous in an appropriate solution space. As the method does not require a compatibility condition between the finite element space, the formulation allows the use of piecewise polynomial spaces of the same approximation order for both the stress and the velocity approximations. A Newton-type iterative method is used to linearize the problem and numerical tests are provided to illustrate the theory.

References:

[1] https://arxiv.org/abs/2305.11635

[2] https://doi.org/10.1002/pamm.201800450

[1]Universität Duisburg-Essen, Fakultät für Mathematik
henrik.schneider@uni-due.de

[2]TU Chemnitz
fleurianne.bertrand@mathematik.tu-chemnitz.de

Notes on session “Scientific Computing”

List of Participants

List of participants

Surname, first name	Abstr.	**from**	**e-mail**
Apel, Thomas	[45]	Germany	thomas.apel@unibw.de
Bammer, Patrick		Austria	patrick.bammer@plus.ac.at
Bauer, Daniel	[23]	Germany	daniel.j.bauer@fau.de
Beier, Johanna	[59]	Germany	beier@ifam.uni-hannover.de
Bertrand, Fleurianne	[57]	Deutschland	fleurianne.bertrand@math.tu-Chemnitz.de
Böhm, Fabian	[24]	Deutschland	fabian.boehm@fau.de
Brockmann, Max	[50]	Germany	brockmann@math.uni-koeln.de
Caforio, Federica		Austria	federica.caforio@uni-graz.at
Castillo, Ernesto	[15]	Chile	ernesto.castillode@usach.cl
Döding, Christian	[10]	Germany	christian.doeding@rub.de
Egger, Herbert		Austria	herbert.egger@ricam.oeaw.ac.at
Endale Mbonde, Nicole		Cameroon	endalenicole@yahoo.fr
Endtmayer, Bernhard	[35]	Germany	endtmayer@ifam.uni-hannover.de
Engertsberger, Felix	[26]	Austria	felix.engertsberger@jku.at
Fesefeldt, Lina	[22]	Germany	lina.fesefeldt@tuhh.de
Firmbach, Max	[61]	Germany	max.firmbach@unibw.de
Franz, Sebastian	[37]	Germany	sebastian.franz@tu-dresden.de
Gander, Martin J.	[20]	Switzerland	martin.gander@unige.ch
Garg, Deepika	[64]	United Kingdom	d.garg@ucl.ac.uk
Goulm, Pierre-Alain	[25]	Germany	goulm@insa-toulouse.fr
Grandits, Thomas		Austria	thomas.grandits@uni-graz.at
Haase, Gundolf		Austria	gundolf.haase@uni-graz.at
Harbrecht, Helmut	[49]	Switzerland	helmut.harbrecht@unibas.ch
Haubold, Tim	[12]	Germany	haubold@ifam.uni-hannover.de
Herzog, Roland	[28]	Germany	roland.herzog@iwr.uni-heidelberg.de
Kaltenbacher, Barbara	[39]	Austria	barbara.kaltenbacher@aau.at
Karabelas, Elias		Österreich	elias.karabelas@uni-graz.at
Keyes, David	[8]	Saudi Arabia	david.keyes@kaust.edu.sa

Surname, first name	Abstr.	**from**	**e-mail**
Köhler, Stephan	[48]	Germany	stephan.koehler@math.tu-freiberg.de
Langer, Ulrich	[46]	Austria	ulanger@numa.uni-linz.ac.at
Lube, Gert	[16]	Germany	lube@math.uni-goettingen.de
Margenberg, Nils	[41]	Germany	margenbn@hsu-hh.de
Matthies, Gunar	[17]	Germany	gunar.matthies@tu-dresden.de
Mokhtari, Reza	[42]	Iran	mokhtari@iut.ac.ir
Monsuur, Harald	[58]	Netherlands	harald.monsuur@hotmail.com
Munch, Peter	[11]	Germany	peter.muench@uni-a.de
Notay, Yvan	[54]	Belgium	yvan.notay@ulb.be
Of, Günther	[34]	Austria	of@tugraz.at
Preuss, Janosch	[36]	United Kingdom	j.preuss@ucl.ac.uk
Ramalho Queiroz Pacheco, Douglas	[13]	Norway	douglas.r.q.pacheco@ntnu.no
Ramezani, Mohadese	[62]	Iran	mohadeseh.ramezani@alumni.iut.ac.ir
Rösch, Arnd		Germany	arnd.roesch@uni-due.de
Saghir, Muhammad Tayyab Bin	[51]	Germany	msaghir@math.tu-dortmund.de
Schneider, Henrik	[65]	Germany	henrik.schneider@uni-due.de
Schöberl, Joachim	[52]	Austria	joachim.schoeberl@tuwien.ac.at
Schönauer, Miriam		Austria	miriam.schoenauer@plus.ac.at
Schussnig, Richard	[29]	Germany	richard.schussnig@uni-a.de
Steinbach, Olaf	[56]	Austria	o.steinbach@tugraz.at
Vianello, Marco		Italy	marcov@math.unipd.it
Winkler, Max	[31]	Germany	max.winkler@mathematik.tu-chemnitz.de
Yadav, Mahima	[32]	Germany	mahima.yadav@rub.de
Zainelabdeen, Marwa	[18]	Germany	marwazinabdeen@gmail.com
Zilk, Philipp	[44]	Germany	philipp.zilk@unibw.de

Institut für Mathematik und wissenschaftliches Rechnen
8010 Graz
https://mathematik.uni-graz.at/en/
www.chemnitz-am.de/cfem2023/