Non-coercive boundary value problems

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Standard tools like the Lax–Milgram lemma or the Céa lemma are based on the coercivity of the bilinear form. A typical assumption for a scalar partial differential equation of second order is $c - \frac{1}{2} \operatorname{div} b > 0$ to ensure coercivity. However, this assumption can be too restrictive when $\operatorname{div} b$ is large.

In this contribution the solution and its finite element approximation are examined for the Neumann boundary value problem in such a case: existence and regularity of the solution in weighted Sobolev spaces, discretization with graded meshes, error estimates in the domain and on the boundary, numerical tests.

The application of these insights to a Neumann optimal control problem is discussed as well. Note that for the analysis of optimal control problems the adjoint problem is used, and the problem under consideration is not self-adjoint.

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