Matrix-free Implementation and Evaluation of the Enriched Galerkin Finite Element Method for the Stokes Problem with Varying Viscosity

Fabian Böhm[1]

The simulation of earth mantle convection with the Finite Element method (FEM) is a challenging task for computational engineers. In the discretization of the underlying mathematical model, the Stokes equation, large numbers of degrees of freedom (DoFs) are involved, which makes matrix-free codes necessary. Furthermore, strong variations and even discontinuities in the viscosity cause the linear system to be highly ill-conditioned, and FEM tend to produce unphysical oscillations. It is unclear, which particular FEM is the single best suited one to use for such problems. Higher order methods offer better approximation properties, like a faster convergence rate but also have a computationally more expensive operator application due to more DoFs and non-zeros. Lower order methods are cheaper in that regard but converge slower. Another criterium is inf-sup stability, which some candidates lack. The classical Taylor-Hood method (P2-P1) is a relevant candidate. A second candidate is the enriched Galerkin method (EG-P0), which uses linear polynomials and a vectorial enrichment for the velocity. It has the fewest DoFs per element of all mixed FEM while being inf-sup stable. In this work, EG-P0 is implemented for the first time with a matrix-free operator application. Additionally, the discrete operator is analyzed regarding the block structure, number of non-zeros per row, and the impact of the enrichment. Furthermore, the computational work for assembling the local matrix and stencil application is quantified. EG-P0 is compared with P2-P1 for analytical 2D and 3D test cases with smoothly varying viscosity, the so called SolVi and MultiSinker test cases. The criteria are how much computational work in flops must be invested to reach a certain error, the convergence rate of the discretization and the solution quality.

[1]Chair for Systemsimulation, FAU
Fabian.boehm@fau.de