Post-processing and improved error estimates of numerical methods for evolutionary systems

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We consider evolutionary systems, i.e. systems of linear partial differential equations arising from the mathematical physics, in the form

$$(\partial M_0 + M_1 + A)U = F$$

where M_0, M_1 are bounded linear self-adjoint operators on a Hilbert space $H = H(\Omega)$ and A is a skew self-adjoint operator on H. Suppose further that there are constants ρ_0 and $\gamma > 0$, such that

$$\rho M_0 + M_1 \geq \gamma$$

for all $\rho \geq \rho_0$. For these systems there exists a general solution theory, see [1, Solution Theory] in exponentially weighted spaces which can be exploited in the analysis of numerical methods.

The numerical method considered is a discontinuous Galerkin method in time combined with a conforming Galerkin method in space. Building on our recent paper [2], we improve some of the results, study the dependence of the numerical solution on the weight-parameter, and consider a reformulation and post-processing of its numerical solution. Numerical simulations support the theoretical findings.

References:

[1] R. Picard: A structural observation for linear material laws in classical mathematical physics, Math. Methods Appl. Sci., 32(14):1768–1803, 2009.

[2] S. Franz, S. Trostorff, M. Waurick: Numerical methods for changing type systems, IMAJNA, 39(2):1009–1038, 2019.

[3] S. Franz: Post-processing and improved error estimates of numerical methods for evolutionary systems, arXiv:2304.12816, 2023.

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