Nonlinear Preconditioning for Implicit Solution of Discretized PDEs

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Abstract Nonlinear preconditioning refers to transforming a nonlinear algebraic system to a form for which Newton-type algorithms have improved success through quicker advance to the domain of quadratic convergence. We place these methods, which go back at least as far as the Additive Schwarz Preconditioned Inexact Newton (ASPIN, 2002), in the context of a proliferation distinguished by being left- or right-sided, multiplicative or additive, non-overlapping or overlapping, and partitioned by field, subdomain, or other criteria. We present the Nonlinear Elimination Preconditioned Inexact Newton (NEPIN, 2021), which is based on a heuristic *bad/good* heuristic splitting of equations and corresponding degrees of freedom. We augment basic forms of nonlinear preconditioning with three features of practical interest: a cascadic identification of the *bad* discrete equation set, an adaptive switchover to ordinary Newton as the domain of convergence is approached, and error bounds on output functionals of the solution. Various nonlinearly stiff algebraic and model PDE problems are considered for insight and we illustrate performance advantage and scaling potential on challenging two-phase flows in porous media.

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