

# Mass-lumping discretization and solvers for distributed elliptic optimal control problems with $L_2$ - regularization

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The purpose of this talk is to investigate the effects of the use of mass-lumping in the finite element discretization of the reduced first-order optimality system arising from a standard tracking-type, distributed elliptic optimal control problem with  $L_2$  regularization. We show that mass-lumping will not affect the  $L_2$  error between the desired state and the computed state, but will lead to a Schur-complement system that allows for a fast matrix-by-vector multiplication. We show that the use of the Schur-Complement Preconditioned Conjugate Gradient method in a nested iteration setting leads to an asymptotically optimal solver with respect to the complexity. Moreover, it is easy to parallelize this solver.

## References:

[1] <http://arxiv.org/abs/2304.14664>

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