# H(div)-conforming dGFEM for turbulent incompressible wall-bounded Navier-Stokes flows

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Three basic problems are relevant for the numerical simulation of turbulent incompressible Navier-Stokes flows: i) "curse of resolution" in space-time for direct numerical simulation (DNS) which critically depends on the Reynolds number, ii) regularity and intermittency of the solution in space-time, iii) occurence of anomalous diffusion in the inviscid limit stemming from the action of fluctuations via the Reynolds stress tensor. These problems call for good solutions w.r.t. computational ressources. We discuss some variants for the numerical modelling of turbulent wall-bounded flows.

H(div)-conforming discontinuous Galerkin-FEM ensure pointwise divergence-free discrete velocities together with pressure-robustness, convection semi-robustness and structure preservation. Moreover they allow a weak imposition of the tangential velocities at a wall and act as implicit large-eddy simulation (ILES). This will be demonstrated for the basic channel flow at bulk Reynolds numbers of $10^3$ ... $10^5$ for the mean streamwise velocity component and the averaged Reynolds stress tensor on slightly anisotropic meshes with less than $10^6$ unknowns, see Schroeder, PhD. Thesis, Goettingen 2019, as opposed to DNS results with up to $10^9$ unknowns.

For very large Reynolds numbers, Hoffman et al. in J. Math. Fluid Mech. (2016) replace the no-slip condition by a Navier-with-slip/friction condition. For a friction coefficient tending to zero, one obtains in the inviscid limit a slip condition. In aviation applications they show that a relatively coarse resolution (with up to $10^6$ ... $10^7$ nodes) is sufficient to obtain reasonable averaged values of lift and drag. We discuss and argue why such approach can be questionable for very large Reynolds numbers. A proper combination of H(div)-conforming dGFEM with Navier-with-slip/friction condition at the wall deserves a stronger consideration. In the inviscid limi, such method with appropriate stabilization can cope with anomalous diffusion, e.g. for the inviscid Taylor-Green vortex, see Fehn, PhD. Thesis, TU Munich 2021.

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