Benchmarking Hybrid Finite Element/Deep Neural Networks and Classical Finite Element Methods

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Accurate flow simulations remain a challenging task. Combining classical finite element approximation techniques with deep neural neworks adds new aspects to the pure numerics-oriented approach and offers potential for further innovations. In this talk we discuss the use of deep neural networks for augmenting classical finite element simulations in fluid-dynamics.

We first establish new benchmark results for the classical DFG-benchmark in 3D using classical finite element simulations with high accuracy. We extend these settings to higher Reynolds numbers and compare two different FEM libraries: Gascoigne3D and deal.II. We compare the computation of drag and lift forces across the two software platforms and show that they are in good agreement.

At high Reynolds numbers, accurate simulations in 3D settings become increasingly difficult, and the classical methods reach their limits. To address this issue, we discuss approaches to connect the finite element method with neural networks. We propose the Deep Neural Network Multigrid Solver, which combines a geometric multigrid solver with a deep neural network to overcome limitations of classical methods. This approach uses classical simulation techniques where their strengths are eminent, such as the efficient representation of a coarse, large-scale flow field. Neural networks are used when a full resolution of the effects does not seem possible or efficient.

We demonstrate the efficiency, generalizability, and scalability of our proposed approach using 3D simulations. Our focus is particularly on issues of stability, generalizability, and error accuracy, and we establish the error accuracy of our proposed method by comparing it with the newly established benchmark results. Overall, our approach offers potential for further innovations in accurate flow simulations.

References:

[1] https://doi.org/10.1016/j.jcp.2022.110983

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