Deep learning approaches based on HDG method for solving some nonlinear elliptic equations

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This talk aims to introduce and analyze two deep neural networks (DNNs) approaches based on the hybridized discontinuous Galerkin (HDG) method for solving some nonlinear elliptic equations. Many known robust numerical methods such as different types of HDG methods have a high dependency on mesh- grid points that makes serious difficulties in problems with complex geometry, especially in high dimensions. Recently, we have constructed two approaches that use artificial neural network approaches to overcome this defect of the classical methods, especially HDG methods. In the first approach, which we called DNN-HDG, after applying the HDG method with a suitable definition of numerical flux and trace, the variational form solutions are approximated directly using the neural networks idea. The second approach, known as DNN-HDG-II, is more compatible with the classical HDG method, in the sense that solutions are considered as linear combinations of the trial functions and then coefficients are approximated using the neural network technique. In this talk, we intend to extend these two efficient and robust methods for solving the following nonlinear elliptic problem

$$\begin{aligned} -\nabla \cdot (\kappa(\mathfrak{u}, \boldsymbol{x})\, \nabla \mathfrak{u}(\boldsymbol{x})) &= \mathfrak{f}(\mathfrak{u}, \boldsymbol{x}), && \boldsymbol{x} \in \Omega \subset \mathbb{R}^d, \\ \mathfrak{u}(\boldsymbol{x}) &= \mathfrak{g}_{_D}(\boldsymbol{x}), && \boldsymbol{x} \in \partial\Omega_D, \\ -\kappa(\mathfrak{u}, \boldsymbol{x})\, \nabla \mathfrak{u}(\boldsymbol{x}) \cdot \boldsymbol{n} &= \mathfrak{g}_{_N}(\boldsymbol{x}), && \boldsymbol{x} \in \partial\Omega_N, \end{aligned} \tag{1}$$

where $d \in \mathbb{N}$ is the spatial dimension, $\boldsymbol{x} = (x_1, \ldots, x_d)^T$, $\boldsymbol{n}$ is the outward unit normal vector, and $\partial\Omega_D$ and $\partial\Omega_N$ are parts of the boundary with Dirichlet and Neumann boundary conditions, respectively. Also, functions κ and $\mathfrak{f}$ are nonlinear terms that are assumed to be in suitable function spaces. We prove that the loss function corresponding to the proposed DNN-HDG methods for solving (1) converges to zero as the mesh step size reduces. Moreover, through some examples, we show that the DNN-HDG methods can efficiently and accurately extract the pattern of the solutions in one, two, and three dimensions. Also, some precious advantages of the DNN-HDG methods compared to the classical HDG methods will be demonstrated, especially for problems with noisy data. Likewise, we demonstrate the ability of the DNN-HDG methods in problems whose exact solutions are not accessible.

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