Optimizing multigrid smoothers for high-order matrix-free FEM computations

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Multigrid methods are among the most competitive solvers for linear systems arising upon discretization of second-order partial differential equations, which occur as subproblems in many application fields in computational science. Within the multigrid algorithm, the most crucial component is usually the smoother, which aims to reduce the high-frequency content in the iteration errors. The observed performance of a smoother depends on the error reduction rate, the possibility for high-performance implementations, and the properties of the underlying hardware itself.

In this presentation, we investigate, in the context of high-order matrix-free FEM computations, point-Jacobi preconditioners and additive Schwarz methods (ASM) based on the fast diagonalization method (FDM) defined on overlapping cell-centered and vertex-star patches. We embed both preconditioners into classical relaxation schemes as well as a Chebyshev iteration and use them as smoothers in the context of p-multigrid. We present novel, highly optimized implementations, which leverage the caches of modern processors by interleaving the work done on the cells in the finite-element discretization and vector operations. We have developed an infrastructure that allows such interleaving in the context of preconditioned conjugate gradient methods [4] and made the implementation freely available via the library deal.II. Here, we show a new application case of this infrastructure.

We conclude our presentation by embedding the developed smoothers into a multigrid scheme to solve Poisson problems on anisotropic meshes. We summarize the results of extensive parameter studies, where we investigate the influence of the number of smoothing steps, the type of decreasing the polynomial degree, the type of preconditioner, the type of Chebyshev polynomials (first or fourth kind), and the type of V-cycle (one- or two-sided). Our results indicate that ASM with FDM, when using the proposed optimizations, can outperform point- Jacobi smoothers on modern CPU-based hardware, especially for meshes with lower-quality and anisotropic elements.

References:

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