# Solving fractional Burgers equations using the Hopf-Cole transformation and local discontinuous Galerkin method

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We study the following time-fractional Burgers equation

$$\begin{aligned} &D_t^\alpha u + \partial_x \left(\frac{u^2}{2}\right) - \mu \Delta u = 0, \qquad (x,t) \in \Omega \times (0,T], \\ &u(\cdot,t)|_{\partial\Omega} = 0, \qquad t \in [0,T], \\ &u(x,0) = u_0(x), \qquad x \in \Omega, \end{aligned}$$

in which $\Omega$ is the spatial domain and $D_t^\alpha u$ is the time fractional derivative of order $\alpha \in (0,1)$, i.e.

$$D_t^\alpha u(\cdot,t) = \frac{1}{\Gamma(2-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial u(\cdot,s)}{\partial s} \,\mathrm{d}s.$$

Recently, Li et al. considered solving the time-fractional Burgers equation using the local discontinuous Galerkin (LDG) method in space and L1 approximation in time. Here, we aim to consider solving this problem with different approaches. By using the Hopf-Cole transformation, the original time-fractional burgers equation is transformed into a subdiffusion equation with the Neumann boundary conditions. Moreover, the solution of the subdiffusion equation sometimes has low-order regularity in time even with smooth initial data. Here, we are concerned with both problems whose solutions have strong regularity and weak singularity. Together with a local discontinuous Galerkin method and a finite difference method (FDM) on a uniform mesh in time for discretizing the spatial and temporal derivatives, respectively, we obtain the numerical solution. Handling the singularities in the typical solution of subdiffusion problems, we establish the FDM on a non-uniform mesh for approximating the Caputo derivative and utilize the LDG method in space direction to derive the fully discrete method. Both numerical schemes have convergence spatial rate $\mathcal{O}(h^{k+1})$ when piecewise polynomials of degree $k$ are used. The Caputo approximations used here are higher order than the L1 formula utilized by Li and his coworkers.

References:

[1] Li, C., Li, D., and Wang, Z. (2021). L1/LDG method for the generalized time-fractional Burgers equation. Mathematics and Computers in Simulation, 187, 357-378

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