

# Finite Element Simulation for Elastic and Plastic Fluids

Muhammad Tayyab Bin Saghir<sup>1</sup> Hogenrich Damanik<sup>2</sup> Stefan Turek<sup>3</sup>

In this study, we present the development of a 2D finite-element solver for the simulation of fluids exhibiting both elastic and plastic constitutive properties. These types of fluids are commonly modeled in the constitutive communities as elastoviscoplastic fluids, where the numerical variables depend on the choice of various constitutive models. Many of these constitutive models involve different numerical frameworks to handle the transition between the solid and liquid phases, such as the Saramito model. Although these strategies offer improved accuracy, the numerical treatment becomes significantly more complex, primarily due to the challenges of identifying the interface between the two phases.

To address these complexities, we propose a similar approach from Emad Chaparian et al. by combining the constitutive models of the Oldroyd-B model and the Papanastasiou model for Bingham fluids within a single Eulerian numerical framework. This strategy, which has recently gained attention in elastoviscoplasticity studies, has demonstrated promising qualitative results. Within this approach, our aim is to approximate the velocity, pressure, and elastic stresses in both space and time. To achieve this, we employ a high-order finite element method for the velocity-stress approximation and a discontinuous pressure element. This specific element pair has proven to be highly effective for accurately capturing the behavior of both the Oldroyd-B and Bingham fluids, including nonlinear viscosity functions.

Our study consists of two main steps. First, we validate each component of the numerical solver individually, ensuring that the approximations and calculations are accurate. This step is crucial to establish the reliability and robustness of our approach. Subsequently, in the second step, we apply the solver to simulate elastoviscoplastic fluid behavior in a porous medium. By investigating the fluid flow and deformation within this specific context, we aim to demonstrate the capabilities and potential of our methodology.

---

<sup>1</sup>Institute of Applied Mathematics (LS III), TU Dortmund University, Vogelpothsweg 87, D-44227 Dortmund, Germany.  
msaghir@math.tu-dortmund.de

<sup>2</sup>Institute of Applied Mathematics (LS III), TU Dortmund University, Vogelpothsweg 87, D-44227 Dortmund, Germany.  
hdamanik@math.uni-dortmund.de

<sup>3</sup>Institute of Applied Mathematics (LS III), TU Dortmund University, Vogelpothsweg 87, D-44227 Dortmund, Germany.  
stefan.turek@mathematik.tu-dortmund.de