Adaptive least-squares space-time finite element methods

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We consider the numerical solution of an operator equation $Bu = f$ by using a least-squares approach. We assume that $B : X \to Y^*$ is an isomorphism, and that $A : Y \to Y^*$ implies a norm in Y, where X and Y are Hilbert spaces. The minimizer of the least-squares functional $\frac{1}{2}\|Bu - f\|^2_{A^{-1}}$, i.e., the solution of the operator equation, is then characterized by the gradient equation with an elliptic and self-adjoint operator $S = B^* A^{-1} B : X \to X^*$. When introducing the adjoint $p = A^{-1}(f - Bu)$ we end up with a saddle point formulation to be solved numerically by using mixed finite element methods. Based on a discrete inf-sup stability condition we derive related a priori error estimates. While the adjoint p is zero by construction, its approximation p_h serves as a posteriori error indicator to drive an adaptive discretization scheme. While this approach can be applied to rather general equations, here we consider second order linear partial differential equations, including the Poisson equation, the heat equation, and the wave equation, in order to demonstrate the potential of this proposed approach which allows us to use almost arbitrary space-time finite element methods for the adaptive solution of time-dependent partial differential equations.

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