An Optimally Convergent Convection-Stabilized Taylor–Hood Finite Element Method for the Oseen Equations

Marwa Zainelabdeen[1] Volker John[2] Christian Merdon[3]

We consider the finite element discretization of the Oseen equations. The LSVS convection stabilization proposed in [1], which is motivated by the underlying vorticity equation (obtained by applying the curl operator to the momentum equation) has the advantage that it leads for the Scott-Vogelius finite element to a pressure-robust method. We extended the LSVS method to the classical Taylor-Hood finite element space which is not pressure-robust. We added the grad-div stabilization to improve the mass conservation. The theoretical result in [2] stating that for large grad-div parameter γ the Taylor-Hood method converges to the Scott–Vogelius method is carried over to the LSVS grad-div stabilized scheme. In addition, by utilizing the already proved $O(h^{k+\frac{1}{2}})$ Scott-Vogelius error estimate from [1], we proved an error estimate for the velocity of the same order for Taylor-Hood finite elements. Numerical studies are performed to test the method and investigate the optimal choice of the LSVS and the grad-div stabilization parameters.

References:

[1] https://epubs.siam.org/doi/10.1137/20M1351230

[2] https://epubs.siam.org/doi/10.1137/100794250

[1]Freie Universität Berlin / Berlin Mathematical School
marwazinabdeen@gmail.com

[2]Freie Universität Berlin / Weierstrass Institute for Applied Analysis and Stochastics, Numerical Mathematics and Scientific Computing
john@wias-berlin.de

[3]Weierstrass Institute for Applied Analysis and Stochastics, Numerical Mathematics and Scientific Computing
Christian.Merdon@wias-berlin.de