Identifying cracks in membranes via their eigenfrequencies - A theoretical and practical approach

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The eigenfrequencies of a vibrating membrane generally depend on its shape. The associated inverse problem which became famous through the work of Kac has been widely discussed in the literature. We look at this problem in the context of cracks. Is it possible to identify a crack in a membrane when its eigenfrequencies are known?

First, we show a theoretical approach to see that it is indeed possible to do so under certain conditions. Then, we present a practical method to confirm our theoretical findings. Concretely, we train a neural network using simulated data to predict the shape of a crack from the corresponding eigenfrequencies. For the simulations we use Isogeometric Analysis, which is well known for its excellent spectral approximation properties. Some of the resulting eigenfunctions have a singularity of type r^ν , thus the corresponding eigenvalues can not be approximated well with uniform refinement procedures. Therefore, we introduce a mesh grading approach based on a singular isogeometric mapping and illustrate optimal convergence order for the eigenfunctions and eigenvalues.

References:

[1] https://doi.org/10.2307/2313748

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