Space-time least-squares finite element methods

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For the numerical solution of an operator equation $Bu = f$ we consider a least-squares approach. We assume that $B : X \to Y^*$ is an isomorphism and that $A : Y \to Y^*$ implies a norm, where X and Y are Hilbert spaces.

Firstly, we assume the differential operator B to be linear. The minimizer of the least-squares functional $\frac{1}{2} \|Bu - f\|_{A^{-1}}$ is then characterized by the gradient equation which involves an elliptic operator $S = B^* A^{-1} B : X \to X^*$. We introduce the adjoint $p = A^{-1}(f - Bu)$ and reformulate the first order optimality system as a saddle point system. Based on a discrete inf-sup condition we discuss related a priori error estimates and use the discrete adjoint p_h to drive an adaptive refinement scheme. Numerical examples will be presented which confirm our theoretical findings. Secondly, we demonstrate how to apply the least-squares approach for the numerical solution of a non linear operator equation $B(u) = f$. We derive the related first order optimality system and discuss its solution via Newton's method. Numerical examples involving the semi-linear heat equation and the quasi-linear Poisson equation will be presented.

Finally, we will conclude with some remarks on future work in this area which needs to be done.

References:

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