

# A unified finite element approach for PDE constrained optimal control problems

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We consider an abstract frame work for the numerical solution of optimal control problems subject to partial differential equations. Examples include not only the distributed control of the Poisson equation, the heat equation, and the wave equation, but also boundary control problems. The approach covers the more standard  $L^2$  setting, and the more recent energy regularization, also including state and control constraints. We discuss regularization and finite element error estimates and derive an optimal relation between the regularization parameter and finite element mesh size in order to balance the accuracy, and the costs. Finally we also discuss the efficient solution of the resulting (non)linear systems of algebraic equations. Numerical examples are given which confirm the theoretical findings.

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