TECHNISCHE UNIVERSITÄT CHEMNITZ

Fakultät für Mathematik

Chemnitz FE-Symposium 2025

Chemnitz FE Symposium

Programme

Collection of abstracts

List of participants

Chemnitz, September 15 - 17, 2025

Scientific Topics:

The symposium is devoted to all aspects of finite elements and related methods for solving partial differential equations.

The topics include (but are not limited to):

- Scientific Computing,
- Mechanics/Applications,
- Inverse Problems,
- Optimization with PDEs,
- Uncertainty Quantification.

This year we particularly encourage talks on:

- Adaptivity,
- Optimal Control,
- Time-dependent Problems.

Invited Speakers:

Konstantinos Chrysafinos (National Technical University of Athens)

Christian Kreuzer (TU Dortmund University)

Ira Neitzel (University of Bonn)

Scientific Committee:

Th. Apel (München), F. Bertrand (Chemnitz), S. Beuchler (Hannover),
O. Ernst (Chemnitz), G. Haase (Graz), H. Harbrecht (Basel),
R. Herzog (Heidelberg), M. Jung (Dresden), U. Langer (Linz),
A. Meyer (Chemnitz), O. Rheinbach (Freiberg), A. Rösch (Duisburg-Essen),
O. Steinbach (Graz), M. Stoll (Chemnitz), M. Winkler (Chemnitz)

Organising Committee:

F. Beer, N. Obszanski, A. Rösch,
N. Windhuis

www.chemnitz-am.de/cfem2025/

Internet Access

The Wolfsburg offers free internet access. Access details can be obtained from the reception.

Catering

The conference fee includes:

- Lunch on all days of the symposium
- Conference Dinner on Monday
- Tea and coffee during breaks

For participants staying at the Wolfsburg there is a breakfast buffet.

Excursions

Two excursions are offered, from which you may choose one:

- **Hike to Landschaftspark Duisburg-Nord** (https://www.landschaftspark.de/en/) with guided tour.
- **Exhibition "Planet Ozean" at Gasometer Oberhausen** (https://www.gasometer.de/en/exhibitions/planet-ozean)

On Tuesday, after the lunch break, both groups meet at the reception desk of Wolfsburg at 1 PM to depart for the excursions.

Both groups will travel by public transport. Tickets will be provided for participants who indicated in the survey that they do not hold a Deutschland-Ticket.

Please wear sturdy shoes and, depending on the weather, bring waterproof clothing.

Both groups are expected to return to Wolfsburg at approximately 6:00 PM in time for dinner.

Programme

Programme for Monday, September 15, 2025

09:00	Opening	Room: Metropole

Discontinuous Galerkin
Chair: Thomas Apel — Room: Metropole

09:05 **Konstantinos Chrysafinos** 8
Discontinuous Galerkin time-stepping schemes for the parabolic p-Laplacian equation

09:55 **Christian Wieners** 9
Localization of the pollution source position with a space-time discontinuous Galerkin method for transport in porous media

10:20 **Gunar Matthies** 10
Higher order hybrid temporal discretizations applied to transient Stokes problems

10:45	*Coffee Break*	-11:15

	Space-Time	Preconditioning
	Chair: Ira Neitzel — Room: Metropole	*Chair*: Gundolf Haase — Room: Arkade
11:15	**Richard Löscher** . . . . . 12 On space-time analysis of parabolic problems on tensor product meshes	**Roland Herzog** . . . . . 17 Preconditioned solution of a magnetostatic optimal control problem
11:40	**Bernhard Endtmayer** . . . . . 13 Anisotropic goal oriented space time adaptivity applied to convection diffusion reaction equation	**Matti Schneider** . . . . . 18 FFT-based preconditioning of X-FEM systems in computational homogenization
12:05	**Ioannis Toulopoulos** . . . . . 14 An anisotropic error estimate for SUPG space-time finite element methods	**Stephan Köhler** . . . . . 19 Multilevel Overlapping Schwarz Preconditioners for Fluid Problems

12:30	*Lunch Break*	-13:30

Programme for Monday, September 15, 2025 (continued)

	Stokes I	Mixed Discretizations
	Stokes I *Chair*: Gunar Matthies Room: Metropole	**Mixed Discretizations** *Chair*: Fleurianne Bertrand Room: Arkade
13:30	**Johannes Pfefferer** . . . 21 Numerical analysis for the Stokes problem with non-homogeneous Dirichlet boundary conditions. Part 1: Weak solutions	**Tugay Dagli** . . . 26 Adaptive Mixed Finite Element Method for Stress-Based Formulations of Eigenvalue Problems
13:55	**Katharina Lorenz** . . . 22 Discretizaion of the Stokes Problem with non-homogeneous Dirichlet boundary conditions. Part 2: Very weak solutions	**Amireh Mousavi** . . . 27 Numerical methods for stochastic non-divergence form elliptic PDEs: a mixed finite element perspective
14:20	**Michael Reichelt** . . . 23 A Space-Time Tensor-Product Finite Element Method for the Stokes System	**Marialetizia Mosconi** . . . 28 New Crouzeix-Raviart spaces of even and variable order
14:55	**Tobias Kaltenbacher** . . . 24 A Simplicial Space-Time Finite Element Method for the Stokes System	**Alexander Linke** . . . 29 Refined stability estimates for mixed problems by exploiting semi norm arguments
15:20	*Coffee Break*	-15:50
	Stokes II *Chair*: Max Winkler Room: Metropole	**Applications** *Chair*: Sven Beuchler Room: Arkade
15:50	**Jonas Glatzel** . . . 31 Quasi-optimal and pressure robust methods for the instationary Stokes equations	**Shahin Heydari** . . . 35 Modeling and numerical study on a collection of strongly coupled chemotaxis-fluid systems
16:15	**Mayssa Mroueh** . . . 32 Non-Polynomial Discontinuous Galerkin Discretization for Navier-Stokes Problems	**Christos Pervolianakis** . . . 36 A priori and a posteriori error analysis of an algebraic flux correction scheme for an optimal control problem
16:40	**Marwa Zainelabdeen** . . . 33 Gradient-robust finite element-finite volume scheme for the compressible Stokes equations	**Manoj Prakash** . . . 37 Computation of stabilization parameters by deep learning
17:30	*Break*	-17:45
17:45	*Before Dinner Talk*	
18:30	*Conference Dinner*	

Programme for Tuesday, September 16, 2025

Optimal Control I
Chair: Arnd Rösch Room: Metropole

10:20 *Coffee Break + Poster Discussion* -10:50

Optimal Control II
Chair: Roland Herzog
Room: Metropole

12:05 *Lunch Break* -13:00

13:00 *Excursion*

18:00 *SC Meeting*

Programme for Wednesday, September 17, 2025

Numerical Analysis
Chair: Olaf Steinbach — Room: Metropole

09:00 **Christian Kreuzer** 47
Accurate error bounds and applications in the error analysis of finite element methods

09:50 **Thomas Apel** 48
Discretization error estimates for Isogeometric Analysis and domains with corners

10:15 *Coffee Break* -10:45

Shape Optimization
Chair: Roland Herzog
Room: Metropole

10:45 **Laura Hetzel** 50
Constrained L^p Approximation of Shape Tensors and its role for the determination of shape gradients

11:10 **Hamdullah Yücel** 51
An Adaptive Finite Element Procedure for Shape Optimization Problems

Fractional PDEs
Chair: Johannes Pfefferer
Room: Scala

10:45 **Reza Mokhtari** 53
Solving Time-Fractional Backward Heat Conduction Problems Using Physics-Informed Kolmogorov-Arnold Networks

11:10 **Felix Beer** 55
The Time-Fractional Cauchy Problem

11:40 Closing — Room: Metropole

12:00 *Lunch*

Collection of Abstracts

Discontinuous Galerkin time-stepping schemes for the parabolic p-Laplacian equation

Konstantinos Chrysafinos[1] Panagiotis Paraschis[2]

We consider fully-discrete schemes for the parabolic p-Laplacian equation. The schemes combine the discontinuous Galerkin time-stepping approach for the temporal discretization with classical conforming finite elements in space. In particular, we are interested in developing a symmetric -Céa Lemma type- error estimate for a suitable quasi-norm, under minimal regularity assumptions on the data. The above estimate leads to error bounds of arbitrary order in space and time provided that the necessary regularity is present, without imposing any restrictions between the temporal and spatial discretization parameters. The symmetric structure of the estimate also leads to various error estimates at partition points as well as for the natural energy $L^p(I;W^{1,p}(\Omega))$ norm. Furthermore, $L^\infty(I;L^2(\Omega))$ stability and error estimates are discussed. Finally, we present some possible extensions of our approach to the error analysis of suitable space-time discontinuous Galerkin schemes.

References:

[1] Error estimates for discontinuous Galerkin time-stepping schemes for the parabolic p-Laplacian: A quasi-norm approach, ESAIM, Math. Model. Numer. Anal, Vol. 59, No 1, 449-485, 2025.

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Localization of the pollution source position with a space-time discontinuous Galerkin method for transport in porous media

Christian Wieners[1] Daniele Corallo[2]

We introduce a parallel adaptive space-time discontinuous Galerkin method for the linear transport equation, where the transport vector is determined from the porous media equation. Given the permeability distribution, in the first step the pressure head and the flux is computed by a mixed approximation of the linear porous media problem. Then, for a given initial pollution distribution the linear transport is approximated by an adaptive DG space-time discretization on a truncated space-time cylinder which turns out to be very efficient since the adaptively refined region is transported with the pollution distribution. The full linear system in space and time is solved with a parallel multigrid method where the stopping criterium for the linear solver is controlled by the convergence of a linear goal functional. Finally we apply this method to solve the inverse problem to reconstruct the initial pollution distribution from measurements of the outflow.

References:

[1] doi = 10.1016/j.camwa.2023.10.031

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Higher order hybrid temporal discretizations applied to transient Stokes problems

Gunar Matthies[1] Lukas Keller[2]

We analyze finite element discretizations of the transient Stokes equations that are based on inf-sup stable finite element pairs for velocity and pressure. A hybrid temporal discretization is applied: a continuous Galerkin–Petrov method is used for the velocity, while the pressure is approximated in a discontinuous manner. We prove optimal convergence orders in space and time for both velocity and pressure. Moreover, a simple postprocessing allows to improve the temporal accuracy of both velocity and pressure by one order. Furthermore, the postprocessed discrete velocity is continuously differentiable, while the postprocessed discrete pressure is continuous. Numerical results support our theoretical findings.

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Notes on session “Discontinuous Galerkin”

On space-time analysis of parabolic problems on tensor product meshes

Richard Löscher[1] Michael Reichelt[2] Olaf Steinbach[3]

In this talk we will focus on parabolic initial-boundary value problems with varying regularity in the initial data. As is known for time stepping methods, rough initial data can lead to oscillations of the finite element solution and needs to be treated using adapted strategies, e.g., geometric warm-up or mixed time stepping schemes. We will discuss space-time variational formulations for the incorporation of the initial data in both a strong and ultra-weak sense in time. Using tensor product meshes, we will derive finite element error estimates that highlight the advantages and limitations of each formulation. Numerical examples will complement the theoretical findings.

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Anisotropic goal oriented space time adaptivity applied to convection diffusion reaction equation

Bernhard Endtmayer[1] Markus Bause[2] Marius Paul Bruchhäuser[3] Nils Margenberg[4] Ioannis Toulopoulos[5]

We present an anisotropic, goal-oriented error estimator based on the Dual Weighted Residual (DWR) method for time-dependent convection-diffusion-reaction equations. By leveraging anisotropic interpolation, the estimator decomposes directional contributions to the error, naturally guiding adaptive mesh refinement in space and time. Stabilization via the SUPG method ensures robustness for high Péclet numbers. Numerical results highlight the efficiency of our method in resolving sharp layers and demonstrate clear advantages over isotropic and globally refined meshes for convection-dominated transport benchmarks.

References:

[1] https://arxiv.org/abs/2504.04951

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An anisotropic error estimate for SUPG space-time finite element methods

Ioannis Toulopoulos[1]

In this talk, we present SUPG space-time FE discretizations on anisotropic meshes for the following initial/boundary - value problem

$$u_t - \operatorname{div}(\varepsilon \nabla_x u) + \boldsymbol{\beta} \cdot \nabla_x u + ru = f \qquad \text{in } Q_T = (0,T) \times \Omega \qquad (1a)$$
$$u = u_\Sigma = 0 \qquad \text{on } \Sigma := \partial\Omega \times [0,T], \qquad (1b)$$
$$u(x,0) = u_0(x) \qquad \text{on } \Sigma_0 := \Omega \times \{0\}, \qquad (1c)$$

where Ω is a bounded cuboid domain in $\mathbb{R}^{d_x}$, with $d_x = 1,2,3$, $T > 0$ a fixed time, $\nabla_x u$ is the spatial gradient of u, $\varepsilon > 0$, $r > 0$ are the diffusion and reaction coefficients, and $\boldsymbol{\beta} := (\beta_x, \beta_y, \beta_z)$ is a constant vector. The main idea of the proposed space-time scheme is to consider the temporal variable t as another spatial variable, let's say, x_{d_x+1}, and to consider u_t as a convection term in the direction x_{d_x+1}. In view of this, we discretize (1) in a unified way in the whole Q_T by applying SUPG finite element methodologies.
The main objective of this talk is to present a posteriori error bounds for the method, evaluated in the same SUPG norm used in the discretization error analysis. The corresponding error estimators are constructed using standard residual terms, including element-wise residuals and interface residuals arising from jumps in the normal fluxes across mesh faces. The development of these estimators relies on suitable anisotropic interpolation estimates, formulated for tensor-product finite-dimensional spaces on rectangular meshes. Notably, these estimates do not require restrictive assumptions on mesh stretching ratios or coordinate alignment. The proposed estimators yield an upper bound on the error measured in the SUPG norm. This is accomplished by analyzing the coercivity properties of the stabilized SUPG bilinear form with respect to that norm. In the final part of the talk, a series of numerical tests are presented to evaluate the properties and effectiveness of the proposed error estimators.
Acknowledgment. (1) The author is grateful to the Department of Informatics, University of Western Macedonia GREECE, for supporting his participation to the Chemnitz Finite Element Symposium Sept. 2025.
(2) The author acknowledges the funding of DAAD-project 57729992, "Goal-oriented AnIsotropic Space- Time Mesh Adaption (AIMASIM)" in the funding program "Programm des projektbezogenen Persone- naustauschs Griechenland ab 2024-2026".

References:

[1] I. Toulopoulos: SUPG space-time scheme on anisotropic meshes for general parabolic equations, Journal of Numerical Mathematics (2025), to appear

[1]Department of Informatics, University of Western Macedonia, GREECE
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[2] M. Bause, M. P. Bruchhäuser, B. Endtmayer, N. Margenberg, I. Toulopoulos, T. Wick: Anisotropic space-time goal-oriented error control and mesh adaptivity for convection-diffusion-reaction equations, under review, 2025

[3] M. Bause, M. P. Bruchhäuser, B. Endtmayer, N. Margenberg, I. Toulopoulos, T. Wick: A posteriori error estimators for SUPG space-time scheme on anisotropic meshes, under review, 2025

Notes on session “Space-Time”

Preconditioned solution of a magnetostatic optimal control problem

Roland Herzog[1] John Pearson[2] Irwin Yousept[3]

We consider the preconditioned iterative solution for the optimal control of magnetostatic equations. The proposed method considers a proper regularization of the saddle-point forward system, featuring a positive-definite structure and preserving the divergence-free condition for the state. On the basis of the resulting optimality system, we develop a specific preconditioner with upper and lower bounds for the eigenvalues with mild dependence on the regularization parameter and the mesh size. This property guarantees the desired robustness of the proposed preconditioner in the limit case, which is as well confirmed by our numerical tests.

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FFT-based preconditioning of X-FEM systems in computational homogenization

Matti Schneider[1] Flavia Gehrig[2]

Multiscale problems are pertinent in engineering applications, as a variety of materials are composed of different materials on a lower scale. To simulate components made of such materials, the theory of homogenizations turns out to be convenient, deriving effective material properties based on resolving corrector problems on volume elements of typically rectangular shape. However, these problems typically involve complex microstructure geometries and non-constant coefficients, which make their resolution numerically challenging.

Computational methods based on the fast Fourier transform (FFT) became increasingly popular in the last decades to resolve homogenization problems. Operating on a regular grid, these approaches side-step the daunting task of generating an interface-conforming mesh. Moreover, FFT methods come with a natural preconditioning strategy based on a constant-coefficient preconditioner, which permits to obtain iteration counts which are bounded independently of the mesh spacing. However, these methods sacrifice accuracy for speed, and lead to effective properties which converge only linearly in the mesh spacing.

The work to be presented is concerned with the extended finite element method (X-FEM), which permits to resolve non-grid-aligned interfaces on a regular grid by augmenting the nodal shape functions with additional, so-called extended degrees of freedom. X-FEM was shown to achieve superior accuracy compared to vanilla regular-grid based methods, but may lead to severely ill-conditioned systems to be solved after discretization. We propose a merger of FFT methods and the X-FEM discretization for homogenization problems, combining the accuracy of X-FEM with the efficiency of FFT-based methods via a resolution independent bound on the iteration count.

This is joint work with F. Gehrig.

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Multilevel Overlapping Schwarz Preconditioners for Fluid Problems

Stephan Köhler[1] Oliver Rheinbach[2]

Additive overlapping Schwarz Methods are domain decomposition methods for the solution of partial differential equations. A second level, the coarse problem, ensures scalability of these methods. One famous coarse space is the generalized Dryja–Smith–Widlund (GDSW) approach. In [2], monolithic overlapping Schwarz preconditioners for saddle point problems were introduced. We present parallel results for the solution of incompressible fluid problems by the combination of the additive overlapping Schwarz solvers implemented in the fast and robust overlapping Schwarz (FROSch) library, which is part of the Trilinos package ShyLU [4,3], and the FEATFLOW library [1].

This work is part of the project StrömungsRaum® - Novel Exascale-Architectures with Heterogeneous Hardware Components for Computational Fluid Dynamics Simulations is funded by the Bundesministerium für Bildung und Forschung (BMBF).

References:

[1] *FEATFLOW*, 2024 (acccessed December 18, 2024).

[2] A. Heinlein, C. Hochmuth, and A. Klawonn: *Monolithic overlapping schwarz domain decomposition methods with gdsw coarse spaces for incompressible fluid flow problems*, SIAM Journal on Scientific Computing, 41 (2019), pp. C291–C316.

[3] A. Heinlein, A. Klawonn, S. Rajamanickam, and O. Rheinbach: *FROSch: A Fast And Robust Overlapping Schwarz Domain Decomposition Preconditioner Based on Xpetra in Trilinos*, Springer International Publishing, 2020, pp. 176–184.

[4] A. Heinlein, A. Klawonn, and O. Rheinbach: *A parallel implementation of a two-level overlapping Schwarz method with energy-minimizing coarse space based on Trilinos*, SIAM J. Sci. Comput., 38 (2016), pp. C713–C747.

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Notes on session “Preconditioning”

Numerical analysis for the Stokes problem with non-homogeneous Dirichlet boundary conditions. Part 1: Weak solutions

Johannes Pfefferer[1] Thomas Apel[2] Katharina Lorenz[3]

This talk addresses the finite element approximation of the Stokes problem

$$\begin{aligned} -\Delta y + \nabla p &= 0 \quad \text{in } \Omega, \\ \nabla \cdot y &= 0 \quad \text{in } \Omega, \\ y &= u \quad \text{on } \Gamma = \partial\Omega. \end{aligned}$$

The domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, is assumed to be polygonal or polyhedral and may be non-convex. The focus is on different strategies for approximating non-homogeneous Dirichlet boundary data $u \in H^t(\Gamma)^d$, and on deriving error estimates in both the energy norm and the $L^2(\Omega)$ norm. We assume $t \geq \frac{1}{2}$ so that a weak solution $(y, p) \in H^1(\Omega)^d \times L^2_0(\Omega)$ exists. The more singular case $t \leq \frac{1}{2}$ will be addressed in Part 2 by Katharina Lorenz.

References:

[1] Th. Apel, K. Lorenz, and J. Pfefferer. Numerical analysis for weak and very weak solutions of the Stokes problem with non-homogenuous boundary condition, in preparation.

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Discretizaion of the Stokes Problem with non-homogeneous Dirichlet boundary conditions. Part 2: Very weak solutions

Katharina Lorenz[1] Thomas Apel[2] Johannes Pfefferer[3]

This talk continues the discussion from Part 1 by Johannes Pfefferer. We consider the Stokes equations

$$\begin{aligned} -\Delta y + \nabla p &= 0 \quad \text{in } \Omega, \\ \nabla \cdot y &= 0 \quad \text{in } \Omega, \\ y &= u \quad \text{on } \Gamma \end{aligned}$$

on a polygonal or polyhedral domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$. In this part, we address the case where the Dirichlet boundary data is not sufficiently regular, specifically when $u \in H^t(\Gamma)^d$ with $t \in [-\frac{1}{2}, \frac{1}{2})$, such that a weak solution cannot be expected. We show how to derive a very weak formulation using the method of transposition. Existence, uniqueness and regularity results are presented.

For the finite element discretization, a regularization method is employed, and the boundary datum is treated using an L^2-projection. Error estimates that show the influence of both the maximal interior angle of the domain and the regularity of the datum are obtained. Numerical experiments are provided to validate the theoretical results.

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A Space-Time Tensor-Product Finite Element Method for the Stokes System

Michael Reichelt[1] Richard Löscher[2] Olaf Steinbach[3]

In this talk, we consider the time-dependent Stokes system in a space-time setting. While the stationary Stokes system yields a symmetric saddle point system, we demonstrate that in a space-time setting this is not the case. However, one can still achieve a system with off diagonal blocks being adjoint to one another by considering the velocity as well as the pressure in anisotropic Sobolev spaces. The presentation will comprise theoretical considerations as well as numerical results. The occurring finite element matrices of the anisotropic setting are realized using a modified Hilbert transform.

References:

[1] https://epub.oeaw.ac.at/?arp=0x003b6473

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A Simplicial Space-Time Finite Element Method for the Stokes System

Tobias Kaltenbacher[1] Olaf Steinbach[2]

In this talk, we consider a space-time finite element method for the time-dependent Stokes system. While classical approaches rely on time-stepping schemes, we propose a fully space-time variational formulation in the Bochner setting. This allows for a unified treatment of spatial and temporal discretization. We further present numerical results on arbitrary and unstructured space-time meshes, which demonstrate the flexibility and effectiveness of the proposed method.

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Notes on session “Stokes I”

Adaptive Mixed Finite Element Method for Stress-Based Formulations of Eigenvalue Problems

Tugay Dagli[1] Fleurianne Bertrand[2]

In this talk, we present a novel three-field mixed finite element method for eigenvalue problems (EVPs) arising in Stokes flow and linear elasticity. To this end, we approximate the Hellinger–Reissner formulation, in which the symmetry of the stress tensor is enforced weakly by introducing a Lagrange multiplier that represents the conservation of angular momentum. We consider a tensor-valued stress field with rows in a Raviart–Thomas space, a discontinuous piecewise-polynomial vector field for the velocity (in Stokes) or displacement (in elasticity), and a continuous piecewise-polynomial space to weakly impose symmetry. This yields a stress–velocity–vorticity formulation for the Stokes EVP and a stress–displacement–rotation formulation for the linear elasticity EVP, both discretized with $RT_k^d - DP_k^d - P_k^{\frac{d(d-1)}{2}}, \quad k \geq 1,\ d \in \{2,3\}$. The formulation offers significant advantages, as it is derived directly from fundamental physical principles, including linear momentum balance, constitutive laws, and—specifically in the Stokes case—mass conservation. Moreover, it enables a direct approximation of the stress field, which is crucial in many applications.

Nevertheless, for eigenfunctions exhibiting limited regularity, the method may experience suboptimal convergence rates under uniform refinement. To address this, we incorporate an adaptive refinement strategy based on efficient and reliable a posteriori error estimators. This adaptive procedure enables the recovery of optimal convergence rates even in the presence of singularities in the solution. Numerical experiments on convex and non-convex domains in both two and three dimensions demonstrate the robustness and efficiency of the proposed adaptive methodology.

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Numerical methods for stochastic non-divergence form elliptic PDEs: a mixed finite element perspective

Amireh Mousavi[1]

In this talk, I will present a numerical framework for solving second-order elliptic partial differential equations in non-divergence form with stochastic coefficients. These arise naturally in models involving uncertainty, where both the diffusion matrix and the forcing term are random.

Our approach is based on a mixed finite element formulation in the physical domain, combined with a stochastic collocation method. A key feature of our method is the incorporation of the vanishing tangential trace condition directly into a mesh-dependent cost functional, avoiding the need to enforce it within the function space itself. This leads to a natural definition of a mesh-dependent norm, which forms the basis for our error analysis.

To handle the stochastic aspect, we use a collocation strategy based on tensor-product orthogonal polynomial nodes, resulting in a decoupled set of deterministic problems that are computationally efficient to solve.

I will also present a priori error estimates for the fully discrete scheme and share numerical results that demonstrate the accuracy and effectiveness of the proposed method, aligning with the theoretical predictions.

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New Crouzeix-Raviart spaces of even and variable order

Marialetizia Mosconi[1] Andrea Bressan[2] Lorenzo Mascotto[3]

The lowest-order Crouzeix–Raviart (CR) element was introduced in 1973 by Crouzeix and Raviart as an elementwise divergence free discretization of the Stokes' equations for space dimensions $d = 2, 3$; The design of CR-type elements of higher polynomial degree p, reveals that their definitions must differ with respect to the parity of p. In this talk, we present new CR-type spaces of even degree p that are spanned by basis functions mimicking those for the standard odd degree case. Compared to the standard even order CR gospel, the present construction allows for the use of nested bases of increasing degree and is particularly suited to design variable order CR methods. We analyze a nonconforming discretization of a two dimensional Poisson problem, which requires a DG-type stabilization. Numerical results are presented, which exhibit the expected convergence rates for the h- and p-versions of the scheme. Finally, the design of variable degree CR global spaces and a corresponding variable order method are discussed.

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Refined stability estimates for mixed problems by exploiting semi norm arguments

Alexander Linke[1] Christian Merdon[2] Nicolas Gauger[3]

Refined stability estimates are derived for classical mixed problems. The novel emphasis is on the importance of semi norms on data functionals, inspired by recent progress on pressure-robust discretizations for the incompressible Navier–Stokes equations. In fact, kernels of these semi norms are shown to be connected to physical regimes in applications and are related to some well-known consistency errors in classical discretizations of mixed problems. Consequently, significantly sharper stability estimates for solutions close to these physical regimes are obtained. Some applications in adaptivity, optimal control and steady and time-dependent problems will be presented.

References:

[1] https://arxiv.org/pdf/2506.11566

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Notes on session “Mixed Discretizations”

Quasi-optimal and pressure robust methods for the instationary Stokes equations

Jonas Glatzel[1] Christian Kreuzer[2]

Pressure robust methods decouple the velocity error from the pressure error, i.e. $\|u - u_h\|$ is independent of $\|p - p_h\|$. This is desirable for instances with small viscosity or if the pressure is difficult to approximate, see [Lin14]. Quasi-optimal methods ensure that the discretization error is bounded by the best error. For a quasi-optimal and pressure robust method we therefore have the Céa-type estimate

$$\|u - u_h\| \leq C \inf_{v_h} \|u - v_h\|.$$

Methods with these characteristics are established for the stationary Stokes equations (e.g. [KVZ21]). Quasi-optimal methods for time dependent problems have been introduced by e.g. [BM97], [Tan14] for the heat equation. In this talk we combine both concepts to derive discretizations for the instationary Stokes equations which are quasi-optimal and pressure robust. Central to the quasi-optimality is a discrete inf-sup-condition for the underlying bilinear form with respect to the considered (discrete) norms. We measure the velocity error in a variety of norms including $L^2(H^1)$- and $L^2(L^2)$-type norms.

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[1] https://epubs.siam.org/doi/abs/10.1137/S0036142994261658

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[3] https://www.sciencedirect.com/science/article/pii/S0045782513002636

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Non-Polynomial Discontinuous Galerkin Discretization for Navier-Stokes Problems

Mayssa Mroueh[1] Jamelot[2] Omnes[3]

We aim to develop a flexible numerical method to simulate the Navier-Stokes equations, with a low kinematic viscosity ν. To this end, we propose a discontinuous Galerkin SIP method, using local exponential basis. This discretization choice is inspired by the exact solution of the one-dimensional advection-diffusion problem.

Concerning the advection-diffusion problem, the basis functions depend on the advection field. We provide 2D simulations showing an improvement in accuracy and a reduction in numerical oscillations, compared to the use of polynomial basis functions. We also provide 2D simulations for Oseen problem. The study of the Navier-Stokes problem is ongoing.

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Gradient-robust finite element-finite volume scheme for the compressible Stokes equations

Marwa Zainelabdeen[1] Christian Merdon[2] Volker John[3] Alexander Linke[4]

We consider the steady compressible Navier–Stokes equations formulated in primitive variables: velocity, pressure, and a non-constant density. A barotropic flow is assumed, where the pressure depends solely on the density via an exponential equation of state.

In this talk, we present a numerical scheme for the compressible Stokes equations that couples a finite element discretization for the momentum equation with a finite volume method for the continuity equation, assuming an isentropic equation of state. The scheme ensures stability, mass conservation, positivity of density, and reserves pressure-robustness when the Mach number goes to zero. The latter property is related to the locking phenomenon observed in incompressible flow at high Reynolds number regimes, which carries over to the compressible setting and is addressed using reconstruction onto H(div)-conforming finite element spaces. The scheme's properties are validated through a range of numerical benchmark problems.

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[1] https://doi.org/10.1016/j.cma.2020.113069

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Notes on session “Stokes II”

Modeling and numerical study on a collection of strongly coupled chemotaxis-fluid systems

Shahin Heydari[1] Thomas Wick[2] Johannes Lankeit[3]

In this work, we study chemotaxis-fluid systems governed by the following general form

$$
\begin{aligned}
&n_t + \mathbf{u}\cdot\nabla n = \Delta n - \nabla\cdot(n\,\mathcal{S}(x,n,c)\,\nabla c) + f(n), && (x,t)\in\Omega\times(0,T),\\
&c_t + \mathbf{u}\cdot\nabla c = \Delta c - h(n,c), && (x,t)\in\Omega\times(0,T),\\
&\mathbf{u}_t + \kappa(\mathbf{u}\cdot\nabla)\mathbf{u} = \Delta\mathbf{u} + \nabla P + n\nabla\Phi + f_{\mathbf{u}}(x,t), && (x,t)\in\Omega\times(0,T),\\
&\nabla\cdot\mathbf{u} = 0, && (x,t)\in\Omega\times(0,T).
\end{aligned}
$$

which models the interaction between motile biological organisms and their chemical signaling in fluid environment. It is motivated by recent analytical studies on chemotaxis-Stokes/Navier-Stokes systems, particularly those involving singular sensitivities, logistic-type source terms, chemical signaling, and general fluid coupling.
We aim to numerically investigate various structurally distinct forms of this system by incorporating logistic source functions $f(n)$ and sensitivity functions $\mathcal{S}(x,n,c)$ in the $n-$equation, both consumption and production signaling mechanisms $h(n,c)$ in the $c-$equation, and modeling the fluid dynamics through either the Stokes or Navier-Stokes in the $u-$equation depending on the variation considered. To capture the complexity of the underlying dynamics, we develop a high-resolution finite element method that incorporates stabilization techniques suited for resolving steep gradients and nonlinear interactions in convection- and chemotaxis-dominated regimes. Numerical simulations are carried out in both two- and three-dimensional domains, supporting analytical and numerical findings reported in the literature.

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A priori and a posteriori error analysis of an algebraic flux correction scheme for an optimal control problem

Christos Pervolianakis[1]

In this talk, we consider an optimal control problem on a bounded domain $\Omega \subset \mathbb{R}^2$, governed by a time-dependent convection–diffusion–reaction equation with pointwise control constraints. We adopt the optimize–then–discretize approach, and the resulting optimality conditions yield a coupled system of two time-dependent convection–diffusion–reaction equations.

It is known that convection–diffusion–reaction equations can develop sharp layers, which pose challenges for standard finite element methods. These layers can cause issues such as spurious oscillations, violating physical properties of the solution. To address these issues, we stabilize the fully discrete scheme derived from the optimality conditions by employing the algebraic flux correction method. The resulting fully discrete scheme based on the backward Euler is nonlinear, and we discuss its well-posedness as well as we derive error estimates. Additionally, we derive a residual-type a posteriori error estimator.

Finally, we provide numerical experiments that validate the theoretical results.

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[1] https://arxiv.org/pdf/2412.21070

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Computation of stabilization parameters by deep learning

Manoj Prakash[1] Petr Knobloch[2]

In convection-dominated regimes, traditional stabilization methods often encounter significant drawbacks: they are either computationally expensive or induce numerical oscillations. In this work, we propose a novel approach that integrates a machine learning model to predict a better stabilization parameter than the standard one used in SUPG. Our methodology employs a neural network that extracts important local features of the problem from a coupled SUPG-Tabata framework, predicting a more appropriate stabilization parameter for the SUPG method. This approach not only aims to reduce computational cost but also mitigates oscillatory inaccuracies, ultimately enhancing the reliability of numerical simulations in convection-dominated environments.

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Notes on session “Applications”

On optimal control problems with quasilinear parabolic PDE and additional inequality constraints

Ira Neitzel[1] Lucas Bonifacius[2] Fabian Hoppe[3] Hannes Meinlschmidt[4]

In this talk, we give an overview about optimal control problems governed by quasilinear parabolic PDEs, subject to additional inequality constraints ranging from plain control bounds to a variety of constraints on the state and its gradient.

We will first provide an overview about existence and regularity results for the state equation and associated challenges. Once a well-defined control-to-state mapping is established, we focus on proving existence of solutions and first order necessary optimality conditions. Since the nonlinear solution operator makes the overall control problem nonconvex, we will also be concerned with second order sufficient optimality conditions for particular cases. Throughout the talk, we will frequently compare the results and challenges to problems with e.g. semilinear state equation.

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Second order analysis for the optimization of time delays

Fredi Tröltzsch[1] Karl Kunisch[2]

For a nonlinear ordinary differential equation with time delay, the differentiation of the solution with respect to the delay is investigated. Special emphasis is laid on the second-order derivative. The results are applied to an associated optimization problem for the time delay. A first- and second-order sensitivity analysis is performed including an adjoint calculus that avoids the second derivative of the state with respect to the delay.

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Adaptive Mixed Finite Element Method for Stress-Based Formulations of Eigenvalue Problems

Fleurianne Bertrand[1] Tugay Dagli[2]

we introduce a novel three-field mixed finite element method for eigenvalue problems (EVPs) in Stokes flow and linear elasticity. Our approach is based on a weakly enforced symmetric Hellinger–Reissner formulation, using a Lagrange multiplier to represent angular momentum conservation. We use a tensor-valued stress field (with rows in a Raviart-Thomas space), a discontinuous piecewise-polynomial field for velocity or displacement, and a continuous piecewise-polynomial space for weak symmetry enforcement. This leads to a stress–velocity–vorticity formulation for Stokes EVPs and a stress–displacement–rotation formulation for elasticity EVPs. Enforcing physical laws like momentum balance, constitutive relations, and mass conservation, this formulation allows direct stress approximation, important in many applications. While uniform refinement may yield suboptimal convergence for low-regularity eigenfunctions, we address this via adaptive refinement guided by reliable a posteriori error estimators. Numerical results in 2D and 3D, on both convex and non-convex domains, confirm the method's efficiency and robustness.

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Notes on session “Optimal Control I”

State-based nested iteration solution of optimal control problems with PDE constraints

Olaf Steinbach[1] Ulrich Langer[2] Richard Löscher[3] Huidong Yang[4]

We consider an abstract framework for the numerical solution of optimal control problems (OCPs) subject to partial differential equations (PDEs). Examples include not only the distributed control of elliptic PDEs such as the Poisson equation discussed in this paper in detail but also parabolic and hyperbolic equations. The approach covers the standard L^2 setting as well as the more recent energy regularization, also including state and control constraints. We discretize OCPs subject to parabolic or hyperbolic PDEs by means of space-time finite elements similar as in the elliptic case. We discuss regularization and finite element error estimates, and derive an optimal relation between the regularization parameter and the finite element mesh size in order to balance the accuracy, and the energy costs for the corresponding control. Finally, we also discuss the efficient solution of the resulting systems of algebraic equations, and their use in a state-based nested iteration procedure that allows us to compute finite element approximations to the state and the control in asymptotically optimal complexity. The numerical results illustrate the theoretical findings quantitatively.

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A stochastic Galerkin method for Dirichlet control problems with random input

Max Winkler[1] Hamdullah Yücel[2]

Considered are optimal Dirichlet boundary control problems governed by partial differential equations with random inputs, in particular, the diffusion and source term may be uncertain. We investigate existence of solutions, optimality conditions and the regularity of the solution.

Furthermore, we propose a numerical scheme using standard finite elements for the spatial discretization and a stochastic Galerkin discretization in the stochastic space to obtain a fully-discrete scheme. We also provide error estimates for that approximations and confirm the validity of these results in numerical experiments. As the resulting linear systems are huge and expensive to solve sophisticated preconditioning techniques are unavoidable. We present a block-diagonal preconditioner and show the robustness with respect to regularization and discretization parameters.

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Adaptive Embedded DG Methods for Optimal Control of Oseen Equations

Harpal Singh[1] Arbaz Khan[2]

In this work, we analyze three hybridized discontinuous Galerkin finite element methods for the control constrained Oseen equations with a non-constant viscosity. This formulation reduces globally coupled degrees of freedom and provides a divergence-conforming and pointwise divergence-free velocity fileld. An optimal convergence of $O(h^2)$ in the diffusion- dominated regime and a sub-optimal $O(h^{3/2})$ in the convection-dominated regime is established for the velocity and control in the L^2-norm for all three schemes in a unified setting using a variational discretization approach for the control. Additionally, the optimal error estimates for the pressure variable are derived. We also derive a new reliable and efficient residual-based a posteriori error estimator for the proposed schemes. Lastly, we conclude with numerical examples in two and three dimensions to validate the performance of proposed schemes.

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Notes on session “Optimal Control II”

Accurate error bounds and applications in the error analysis of finite element methods

Christian Kreuzer[1]

An error bound is typically an inequality between two seminorms and we may call such an error bound accurate whenever the two seminorms are equivalent. We introduce the abstract concept illustrated by well-known estimates for linear interpolation operators $\mathcal{I}_h$. For linear Lagrange elements and regular target function u, we typically have interpolation estimates of the form

$$\inf_{v_h} \|u - v_h\|_1 \leq C\|u - \mathcal{I}_h u\|_1 \leq Ch|u|_2.$$

Clearly, all three quantities are semi-norms. The first inequality is accurate for the Scott-Zhang interpolation but not for the Lagrange interpolation. Although the kernels coincide, the second inequality cannot be accurate as it requires more regularity than the other semi-norms.

After a basic yet fundamental discussion on semi-norms and their relations to accurate bounds, we shall present and survey recent a priori and a posteriori accurate bounds in the error analysis of finite element methods.

The presentation gives an overview of current developments in the error analysis of finite element methods, which were partly developed also together with or from Andreas Veeser and Pietro Zanotti.

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Discretization error estimates for Isogeometric Analysis and domains with corners

Thomas Apel[1] Philipp Zilk[2]

Isogeometric analysis (IGA) enables exact representations of computational geometries and higher-order approximation of PDEs. In non-smooth domains, however, singularities near corners limit the effectiveness of IGA, since standard methods typically fail to achieve optimal convergence rates. These constraints can be addressed through local mesh refinement, but existing approaches require breaking the tensor-product structure of splines, which leads to increased implementation complexity.

This work introduces a novel local refinement strategy based on a polar parameterization, in which one edge of the parametric square is collapsed into the corner. By grading the standard mesh toward the collapsing edge, the desired locality near the singularity is obtained while maintaining the tensor-product structure.

Polar parameterizations, however, suffer from a lack of regularity at the polar point, making existing standard isogeometric approximation theory inapplicable. To address this, a new framework is developed for deriving error estimates on polar domains with corners. This involves the construction of polar function spaces on the parametric domain and a modified projection operator onto the space of C^0-smooth polar splines. The theoretical results are verified by numerical experiments confirming both the accuracy and efficiency of the proposed approach.

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Notes on session “Numerical Analysis”

Constrained L^p Approximation of Shape Tensors and its role for the determination of shape gradients

Laura Hetzel[1] Gerhard Starke[2]

A crucial issue in numerically solving PDE-constrained shape optimization problems is avoiding mesh degeneracy. Recently, there were two suggested approaches to tackle this problem: (i) departing from the Hilbert space towards the Lipschitz topology approximated by W^{1,p^*} with $p^* > 2$ and (ii) using the symmetric rather than the full gradient to define a norm.

In this talk we will discuss an approach that allows to combine both. It is based on our earlier work [2] on the L^p approximation of the shape tensor of Laurain & Sturm [1]. There we have shown that W^{1,p^*} shape gradients can be determined as Lagrange multipliers of this L^p approximation problem. We extend this by adding a symmetry constraint to the derived L^p least mean approximation problem and show that the distance measured in a suitably weighted L^p-norm is equal to the dual norm of the shape derivative with respect to the L^{p^*}-norm associated with the linear elastic strain of the deformation. The resulting L^p least mean problem can be viewed as a generalization of a constrained first-order least squares formulation. In addition, as in the case without symmetry constraint, it turns out that the Lagrange multiplier associated with the divergence constraint is the direction of the steepest descent, but now with respect to the norm defined by the symmetric gradient. This provides a way to compute shape gradients in W^{1,p^*} with respect to this elasticity type norm.

The discretization of the resulting least mean problem can be done by the PEERS element and its three-dimensional counterpart. We will illustrate the advantages of this approach by computational results of some common shape optimization problems.

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An Adaptive Finite Element Procedure for Shape Optimization Problems

Hamdullah Yücel[1] Oğuz Han Altıntaş[2]

The numerical investigation of shape optimization problems is both computationally and theoretically more complex than solving direct analysis problems using the finite element method. The limitations are mainly due to the involvement of both structural analysis and optimization processes. To overcome these numerical challenges, we present an adaptive algorithm for solving the shape optimization problem associated with the compliance minimization objective with penalized volume fraction. This approach takes into account not only the errors due to the discretization of the constraining PDE, in particular the linear elasticity system, but also the errors due to the discretization of the deformation bilinear form that provides a descent direction. The proposed adaptive procedure is tested on the well-known compliance minimization problem of cantilever beam to illustrate the superiority of the adaptive procedure compared to the fixed mesh refinement.

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Notes on session “Shape Optimization”

Solving Time-Fractional Backward Heat Conduction Problems Using Physics-Informed Kolmogorov-Arnold Networks

Reza Mokhtari[1] Maryam Mohammadi[2]

Artificial neural networks (ANNs) are powerful tools for approximating complex functions, with recent applications in solving challenging partial differential equations (PDEs). In particular, inverse problems, which involve recovering hidden information (e.g., initial conditions, parameters, or source terms) from partial or indirect measurements, remain a critical yet notoriously ill-posed class of problems. Traditional numerical methods such as FDM and FEM suffer from instability, requiring significant regularization techniques or extensive data. Neural networks offer a promising alternative, leveraging data and physics to overcome these limitations, particularly with noisy or sparse data. The Physics-Informed Neural Network (PINN) framework integrates governing differential equations and physical laws into a neural network training loss function. We consider the following time-fractional backward heat conduction problem (TF-BHCP), an ill-posed inverse problem relevant to non-destructive testing and modeling anomalous diffusion

$$\mathrm{D}_t^\alpha u(\boldsymbol{x},t) = \kappa(t)\,\Delta u(\boldsymbol{x},t) + f(\boldsymbol{x},t,u), \qquad (\boldsymbol{x},t) \in \Omega \times (0,T),$$

where $T > 0$ is the final time, $\Omega \subset \mathbb{R}^d$, $d = 1,2,3$, is a connected and bounded domain, f is a given source term, Δ is the Laplace operator, $\kappa \in \mathrm{C}([0,T])$ is a positive time-dependent thermal diffusivity factor, and D_t^α denote the Caputo fractional derivative of order $\alpha \in (0,1)$ which is defined as $\mathrm{D}_t^\alpha u(\cdot,t) := \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{u_t(\cdot,s)}{(t-s)^\alpha} \mathrm{d}s$. We need to have the overdetermined condition $u(\boldsymbol{x},T) = g(\boldsymbol{x})$ for $\boldsymbol{x} \in \Omega$ as well as some suitable Dirichlet or Neumann boundary conditions. Here, g is a known function. The existence and uniqueness conditions, as well as the improperly posed nature of the TF-BHCP, have been discussed in the literature. We propose a PINN-based framework that utilizes Kolmogorov-Arnold Networks (KANs) as the underlying neural architecture. KANs are a newly introduced class of networks based on Kolmogorov's superposition theorem, providing structured and interpretable models with improved approximation capabilities. By embedding KANs within the PINN framework, we design a physics- informed loss function that incorporates the residual of the time-fractional PDE, final-time conditions (due to the backward nature of the problem), and boundary constraints. We also provide a theoretical guarantee by demonstrating that, under suitable assumptions, the loss function converges to zero, and the KAN-PINN solutions converge weakly to a weak solution of the TF-BHCP despite the problem's ill-posedness. Finally, we validate our framework through extensive numerical experiments, demonstrating that it surpasses traditional methods, such as FDM, in terms of accuracy, noise robustness, and adaptability to various frac-

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The Time-Fractional Cauchy Problem

Felix Beer[1]

Time-fractional PDEs have proven useful in various situation where classical models fail to capture memory effects, anomalous diffusion, or power-law dynamics. Applications arise across physics and engineering, biology, finance and economics. This talk will provide a first introduction to time-fractional PDEs, with emphasis on the abstract Cauchy problem of the form $\partial^\alpha u(t) = f(t, u(t)) - Au(t)$, where f satisfies a second variable local Lipschitz condition and A being a linear - possibly unbounded - Operator, generating a C_0-semigroup.

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