Discretization error estimates for Isogeometric Analysis and domains with corners

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Isogeometric analysis (IGA) enables exact representations of computational geometries and higher-order approximation of PDEs. In non-smooth domains, however, singularities near corners limit the effectiveness of IGA, since standard methods typically fail to achieve optimal convergence rates. These constraints can be addressed through local mesh refinement, but existing approaches require breaking the tensor-product structure of splines, which leads to increased implementation complexity.

This work introduces a novel local refinement strategy based on a polar parameterization, in which one edge of the parametric square is collapsed into the corner. By grading the standard mesh toward the collapsing edge, the desired locality near the singularity is obtained while maintaining the tensor-product structure.

Polar parameterizations, however, suffer from a lack of regularity at the polar point, making existing standard isogeometric approximation theory inapplicable. To address this, a new framework is developed for deriving error estimates on polar domains with corners. This involves the construction of polar function spaces on the parametric domain and a modified projection operator onto the space of C^0-smooth polar splines. The theoretical results are verified by numerical experiments confirming both the accuracy and efficiency of the proposed approach.

References:

[1] https://doi.org/10.48550/arXiv.2505.10095

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