Adaptive Mixed Finite Element Method for Stress-Based Formulations of Eigenvalue Problems

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In this talk, we present a novel three-field mixed finite element method for eigenvalue problems (EVPs) arising in Stokes flow and linear elasticity. To this end, we approximate the Hellinger–Reissner formulation, in which the symmetry of the stress tensor is enforced weakly by introducing a Lagrange multiplier that represents the conservation of angular momentum. We consider a tensor-valued stress field with rows in a Raviart–Thomas space, a discontinuous piecewise-polynomial vector field for the velocity (in Stokes) or displacement (in elasticity), and a continuous piecewise-polynomial space to weakly impose symmetry. This yields a stress–velocity–vorticity formulation for the Stokes EVP and a stress–displacement–rotation formulation for the linear elasticity EVP, both discretized with $RT_k^d - DP_k^d - P_k^{\frac{d(d-1)}{2}}, \quad k \geq 1,\ d \in \{2,3\}$. The formulation offers significant advantages, as it is derived directly from fundamental physical principles, including linear momentum balance, constitutive laws, and—specifically in the Stokes case—mass conservation. Moreover, it enables a direct approximation of the stress field, which is crucial in many applications.

Nevertheless, for eigenfunctions exhibiting limited regularity, the method may experience suboptimal convergence rates under uniform refinement. To address this, we incorporate an adaptive refinement strategy based on efficient and reliable a posteriori error estimators. This adaptive procedure enables the recovery of optimal convergence rates even in the presence of singularities in the solution. Numerical experiments on convex and non-convex domains in both two and three dimensions demonstrate the robustness and efficiency of the proposed adaptive methodology.

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