Modeling and numerical study on a collection of strongly coupled chemotaxis-fluid systems

Shahin Heydari[1] Thomas Wick[2] Johannes Lankeit[3]

In this work, we study chemotaxis-fluid systems governed by the following general form

$$
\begin{aligned}
&n_t + \mathbf{u}\cdot\nabla n = \Delta n - \nabla\cdot(n\,\mathcal{S}(x,n,c)\,\nabla c) + f(n), && (x,t)\in\Omega\times(0,T),\\
&c_t + \mathbf{u}\cdot\nabla c = \Delta c - h(n,c), && (x,t)\in\Omega\times(0,T),\\
&\mathbf{u}_t + \kappa(\mathbf{u}\cdot\nabla)\mathbf{u} = \Delta\mathbf{u} + \nabla P + n\nabla\Phi + f_{\mathbf{u}}(x,t), && (x,t)\in\Omega\times(0,T),\\
&\nabla\cdot\mathbf{u} = 0, && (x,t)\in\Omega\times(0,T).
\end{aligned}
$$

which models the interaction between motile biological organisms and their chemical signaling in fluid environment. It is motivated by recent analytical studies on chemotaxis-Stokes/Navier-Stokes systems, particularly those involving singular sensitivities, logistic-type source terms, chemical signaling, and general fluid coupling.
We aim to numerically investigate various structurally distinct forms of this system by incorporating logistic source functions $f(n)$ and sensitivity functions $\mathcal{S}(x,n,c)$ in the $n-$equation, both consumption and production signaling mechanisms $h(n,c)$ in the $c-$equation, and modeling the fluid dynamics through either the Stokes or Navier-Stokes in the $u-$equation depending on the variation considered. To capture the complexity of the underlying dynamics, we develop a high-resolution finite element method that incorporates stabilization techniques suited for resolving steep gradients and nonlinear interactions in convection- and chemotaxis-dominated regimes. Numerical simulations are carried out in both two- and three-dimensional domains, supporting analytical and numerical findings reported in the literature.

[1]Leibniz University Hannover, Institute of Applied Mathematics
heydari@ifam.uni-hannover.de

[2]Leibniz University Hannover, Institute of Applied Mathematics
wick@ifam.uni-hannover.de

[3]Leibniz University Hannover, Institute of Applied Mathematics
lankeit@ifam.uni-hannover.de