Accurate error bounds and applications in the error analysis of finite element methods

Christian Kreuzer[1]

An error bound is typically an inequality between two seminorms and we may call such an error bound accurate whenever the two seminorms are equivalent. We introduce the abstract concept illustrated by well-known estimates for linear interpolation operators $\mathcal{I}_h$. For linear Lagrange elements and regular target function u, we typically have interpolation estimates of the form

$$\inf_{v_h} \|u - v_h\|_1 \leq C\|u - \mathcal{I}_h u\|_1 \leq Ch|u|_2.$$

Clearly, all three quantities are semi-norms. The first inequality is accurate for the Scott-Zhang interpolation but not for the Lagrange interpolation. Although the kernels coincide, the second inequality cannot be accurate as it requires more regularity than the other semi-norms.

After a basic yet fundamental discussion on semi-norms and their relations to accurate bounds, we shall present and survey recent a priori and a posteriori accurate bounds in the error analysis of finite element methods.

The presentation gives an overview of current developments in the error analysis of finite element methods, which were partly developed also together with or from Andreas Veeser and Pietro Zanotti.

[1]TU Dortmund
christian.kreuzer@tu-dortmund.de