Discretizaion of the Stokes Problem with non-homogeneous Dirichlet boundary conditions. Part 2: Very weak solutions

Katharina Lorenz[1] Apel[2] Pfefferer[3]

This talk continues the discussion from Part 1 by Johannes Pfefferer. We consider the Stokes equations

$$\begin{aligned} -\Delta y + \nabla p &= 0 \quad \text{in } \Omega, \\ \nabla \cdot y &= 0 \quad \text{in } \Omega, \\ y &= u \quad \text{on } \Gamma \end{aligned}$$

on a polygonal or polyhedral domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$. In this part, we address the case where the Dirichlet boundary data is not sufficiently regular, specifically when $u \in H^t(\Gamma)^d$ with $t \in [-\frac{1}{2}, \frac{1}{2})$, such that a weak solution cannot be expected. We show how to derive a very weak formulation using the method of transposition. Existence, uniqueness and regularity results are presented.

For the finite element discretization, a regularization method is employed, and the boundary datum is treated using an L^2-projection. Error estimates that show the influence of both the maximal interior angle of the domain and the regularity of the datum are obtained. Numerical experiments are provided to validate the theoretical results.

[1]Bundeswehr Universität München, Institut für Mathematik und Computergestützte Simulation
katharina.lorenz@unibw.de

[2]Bundeswehr Universität München, Institut für Mathematik und Computergestützte Simulation
thomas.apel@unibw.de

[3]Bundeswehr Universität München, Institut für Mathematik und Computergestützte Simulation
johannes.pfefferer@unibw.de