Solving Time-Fractional Backward Heat Conduction Problems Using Physics-Informed Kolmogorov-Arnold Networks

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Artificial neural networks (ANNs) are powerful tools for approximating complex functions, with recent applications in solving challenging partial differential equations (PDEs). In particular, inverse problems, which involve recovering hidden information (e.g., initial conditions, parameters, or source terms) from partial or indirect measurements, remain a critical yet notoriously ill-posed class of problems. Traditional numerical methods such as FDM and FEM suffer from instability, requiring significant regularization techniques or extensive data. Neural networks offer a promising alternative, leveraging data and physics to overcome these limitations, particularly with noisy or sparse data. The Physics-Informed Neural Network (PINN) framework integrates governing differential equations and physical laws into a neural network training loss function. We consider the following time-fractional backward heat conduction problem (TF-BHCP), an ill-posed inverse problem relevant to non-destructive testing and modeling anomalous diffusion

$$\mathrm{D}_t^{\alpha} u(\boldsymbol{x},t) = \kappa(t)\,\Delta u(\boldsymbol{x},t) + f(\boldsymbol{x},t,u), \qquad (\boldsymbol{x},t) \in \Omega \times (0,T),$$

where $T > 0$ is the final time, $\Omega \subset \mathbb{R}^d$, $d = 1,2,3$, is a connected and bounded domain, f is a given source term, Δ is the Laplace operator, $\kappa \in \mathrm{C}([0,T])$ is a positive time-dependent thermal diffusivity factor, and D_t^{α} denote the Caputo fractional derivative of order $\alpha \in (0,1)$ which is defined as $\mathrm{D}_t^{\alpha} u(\cdot,t) := \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{u_t(\cdot,s)}{(t-s)^{\alpha}} \mathrm{d}s$. We need to have the overdetermined condition $u(\boldsymbol{x},T) = g(\boldsymbol{x})$ for $\boldsymbol{x} \in \Omega$ as well as some suitable Dirichlet or Neumann boundary conditions. Here, g is a known function. The existence and uniqueness conditions, as well as the improperly posed nature of the TF-BHCP, have been discussed in the literature. We propose a PINN-based framework that utilizes Kolmogorov-Arnold Networks (KANs) as the underlying neural architecture. KANs are a newly introduced class of networks based on Kolmogorov's superposition theorem, providing structured and interpretable models with improved approximation capabilities. By embedding KANs within the PINN framework, we design a physics- informed loss function that incorporates the residual of the time-fractional PDE, final-time conditions (due to the backward nature of the problem), and boundary constraints. We also provide a theoretical guarantee by demonstrating that, under suitable assumptions, the loss function converges to zero, and the KAN-PINN solutions converge weakly to a weak solution of the TF-BHCP despite the problem's ill-posedness. Finally, we validate our framework through extensive numerical experiments, demonstrating that it surpasses traditional methods, such as FDM, in terms of accuracy, noise robustness, and adaptability to various fractional orders.

References:

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