Numerical methods for stochastic non-divergence form elliptic PDEs: a mixed finite element perspective

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In this talk, I will present a numerical framework for solving second-order elliptic partial differential equations in non-divergence form with stochastic coefficients. These arise naturally in models involving uncertainty, where both the diffusion matrix and the forcing term are random.
Our approach is based on a mixed finite element formulation in the physical domain, combined with a stochastic collocation method. A key feature of our method is the incorporation of the vanishing tangential trace condition directly into a mesh-dependent cost functional, avoiding the need to enforce it within the function space itself. This leads to a natural definition of a mesh-dependent norm, which forms the basis for our error analysis.
To handle the stochastic aspect, we use a collocation strategy based on tensor-product orthogonal polynomial nodes, resulting in a decoupled set of deterministic problems that are computationally efficient to solve.
I will also present a priori error estimates for the fully discrete scheme and share numerical results that demonstrate the accuracy and effectiveness of the proposed method, aligning with the theoretical predictions.

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