FFT-based preconditioning of X-FEM systems in computational homogenization

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Multiscale problems are pertinent in engineering applications, as a variety of materials are composed of different materials on a lower scale. To simulate components made of such materials, the theory of homogenizations turns out to be convenient, deriving effective material properties based on resolving corrector problems on volume elements of typically rectangular shape. However, these problems typically involve complex microstructure geometries and non-constant coefficients, which make their resolution numerically challenging.

Computational methods based on the fast Fourier transform (FFT) became increasingly popular in the last decades to resolve homogenization problems. Operating on a regular grid, these approaches side-step the daunting task of generating an interface-conforming mesh. Moreover, FFT methods come with a natural preconditioning strategy based on a constant-coefficient preconditioner, which permits to obtain iteration counts which are bounded independently of the mesh spacing. However, these methods sacrifice accuracy for speed, and lead to effective properties which converge only linearly in the mesh spacing.

The work to be presented is concerned with the extended finite element method (X-FEM), which permits to resolve non-grid-aligned interfaces on a regular grid by augmenting the nodal shape functions with additional, so-called extended degrees of freedom. X-FEM was shown to achieve superior accuracy compared to vanilla regular-grid based methods, but may lead to severely ill-conditioned systems to be solved after discretization. We propose a merger of FFT methods and the X-FEM discretization for homogenization problems, combining the accuracy of X-FEM with the efficiency of FFT-based methods via a resolution independent bound on the iteration count.

This is joint work with F. Gehrig.

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