State-based nested iteration solution of optimal control problems with PDE constraints

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We consider an abstract framework for the numerical solution of optimal control problems (OCPs) subject to partial differential equations (PDEs). Examples include not only the distributed control of elliptic PDEs such as the Poisson equation discussed in this paper in detail but also parabolic and hyperbolic equations. The approach covers the standard L^2 setting as well as the more recent energy regularization, also including state and control constraints. We discretize OCPs subject to parabolic or hyperbolic PDEs by means of space-time finite elements similar as in the elliptic case. We discuss regularization and finite element error estimates, and derive an optimal relation between the regularization parameter and the finite element mesh size in order to balance the accuracy, and the energy costs for the corresponding control. Finally, we also discuss the efficient solution of the resulting systems of algebraic equations, and their use in a state-based nested iteration procedure that allows us to compute finite element approximations to the state and the control in asymptotically optimal complexity. The numerical results illustrate the theoretical findings quantitatively.

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