

A stochastic Galerkin method for Dirichlet control problems with random input

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Considered are optimal Dirichlet boundary control problems governed by partial differential equations with random inputs, in particular, the diffusion and source term may be uncertain. We investigate existence of solutions, optimality conditions and the regularity of the solution.

Furthermore, we propose a numerical scheme using standard finite elements for the spatial discretization and a stochastic Galerkin discretization in the stochastic space to obtain a fully-discrete scheme. We also provide error estimates for that approximations and confirm the validity of these results in numerical experiments. As the resulting linear systems are huge and expensive to solve sophisticated preconditioning techniques are unavoidable. We present a block-diagonal preconditioner and show the robustness with respect to regularization and discretization parameters.

References:

[1] <https://arxiv.org/abs/2506.11479>

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