TECHNISCHE UNIVERSITÄT
CHEMNITZ

Fakultät für Mathematik

Chemnitz FE-Symposium 2026

Chemnitz
FE
Symposium

Programme

Collection of abstracts

List of participants

Chemnitz, September 07 - 09, 2026

Scientific Topics:

The symposium is devoted to all aspects of finite elements and related methods for solving partial differential equations.

The topics include (but are not limited to):

- Scientific Computing,
- Mechanics/Applications,
- Inverse Problems,
- Optimization with PDEs,
- Uncertainty Quantification.

This year we particularly encourage talks on:

- Porous Media,
- Uncertainty Quantification,
- Computational Machine Learning,
- Curved Boundaries.

Invited Speakers:

Inga Berre (University of Bergen)

Gianluigi Rozza (SISSA Triest)

Ridgway Scott (University of Chicago)

Cecilia Pagliantini (University of Pisa)

Scientific Committee:

Th. Apel (München), F. Bertrand (Chemnitz), S. Beuchler (Hannover),
O. Ernst (Chemnitz), G. Haase (Graz), H. Harbrecht (Basel),
R. Herzog (Heidelberg), U. Langer (Linz), A. Meyer (Chemnitz),
O. Rheinbach (Freiberg), A. Rösch (Duisburg-Essen), O. Steinbach (Graz),
M. Stoll (Chemnitz), M. Winkler (Chemnitz)

Organising Committee:

F. Bertrand, T. Dagli, A.-K. Glanzberg,
Z. Kassali, K. Seidel, M. Winkler
www.chemnitz-am.de/cfem2026/

Internet Access

The c/o56 hotel offers free internet access. Access details can be obtained from the reception.

Food

The conference fee includes:

- Lunch on all days of the symposium
- Dinner on Monday and Tuesday
- Tea and coffee during breaks

For participants staying at the c/o56 hotel there is a breakfast buffet.

Excursion and Social Program

Participants arriving on **Sunday** will find several opportunities for dinner in the vicinity of the conference hotel. The hotel's own restaurant, *Lounge56*, offers an à la carte dinner menu and is open daily in the evening. Just a short walk from the hotel, the historic *Altes Viertel* offers several restaurants, including the Italian restaurant *Amore Mio*, the traditional German restaurant *An der Schlossmühle*, and the Greek restaurant *Ilios*. Further restaurants and bars can be found in Chemnitz city centre, approximately a 30-minute walk from the hotel.

On **Monday evening**, we will have dinner at the *Miramar*, Chemnitz's largest beer garden, beautifully located on the Schloßberg overlooking the Schlossteich.

On **Tuesday**, we will visit the *Einsiedler Brauhaus*, a traditional brewery located in the Einsiedel district on the outskirts of Chemnitz. We will travel there by public transport. The programme includes a guided brewery tour with beer tasting, followed by a cold buffet featuring a selection of meats, cheeses, and bread.

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Collection of Abstracts

Reduced order methods meet machine learning for accelerating computational fluid dynamics: focus on turbulent parametric flows

Gianluigi Rozza[1]

Computational Fluid Dynamics (CFD) is a cornerstone for the analysis of complex flow phenomena.

However, the computational cost of high-fidelity simulations often restricts their use in many-query scenarios such as design optimization, uncertainty quantification, and parametric analysis.

Reduced Order Models (ROMs) alleviate this limitation by constructing low- dimensional surrogates that retain the dominant flow dynamics at a significantly reduced computational cost [1]. This work presents a set of complementary data-driven strategies that combine reduced-order modeling and scientific machine learning to improve the accuracy, robustness, and generalization capabilities of ROMs for CFD applications.

A first contribution focuses on non-intrusive ROMs and introduces a space- dependent aggregation framework in which multiple surrogate models—obtained from different combinations of dimensionality reduction techniques, regression methods, or turbulence models—are locally combined through spatially varying convex weights learned from data. This approach enhances predictive accuracy in challenging benchmark problems, including transonic airfoil flows [2].

A second contribution addresses intrusive ROMs through parametric closure modeling, where a deep operator network is trained to learn reduced correction operators that compensate for unresolved dynamics, enabling reliable generalization across the parameter space [3]. Finally, a hybrid reduced-order modeling framework is developed for high-Reynolds-number fluid–structure interaction problems in an Arbitrary Lagrangian–Eulerian setting. The approach combines POD–Galerkin projection for the fluid equations with machine-learning- based models for the turbulent eddy viscosity and reduced interpolation techniques for mesh motion, enabling accurate and efficient simulation of flow-induced vibration phenomena [4].

Overall, the proposed methodologies advance hybrid reduced-order modeling by unifying surrogate aggregation, operator learning, and data-driven turbulence closure within a flexible and scalable framework, providing fast and reliable CFD predictions for complex, turbulent, and parameter-dependent systems.

References:

[1] Ivagnes, A., Khamlich, M., Siena, P., Rozza, G. Reduced Order Modeling in Computational Fluid Dynamics: An Overview of Methods and Applications. In International Conference on Emerging Technologies in Computational Science for Industry, Sustainability and Innovation (pp. 1-20).

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Cham: Springer Nature Switzerland. (2023, May).

[2] Ivagnes, A., Tonicello, N., Cinnella, P., Rozza, G. Enhancing non-intrusive reduced-order models with space-dependent aggregation methods. Acta Mechanica, 1-30. (2024).

[3] Ivagnes, A., Stabile, G., Rozza, G. Data-driven Closure Strategies for Parametrized Reduced Order Models via Deep Operator Networks. arXiv preprint arXiv:2505.17305. (2025).

[4] Ngan, V. N., Stabile, G., Mola, A., Rozza, G. A hybrid reduced-order model for segregated fluid-structure interaction solvers in an ALE approach at high Reynolds number. Computers & Mathematics with Applications, 180, 299-321. (2025).

Numerical simulations of wetting on biomembranes

Timo Höllein[1] Sebastian Aland[2]

After the discovery of biomolecular condensates in biological cells, research in this field has increased dramatically. Cells use those condensate droplets as well as membranes to structure their interior. The interaction between membrane and droplet can lead to topological changes in cells like fission or fusion of membranes, which are little understood. In this application-oriented talk, we present a first numerical model, discretized using the finite element method, for simulating the wetting of biomembranes by condensates, including scenarios involving topological changes. The model uses a ternary phase-field approach and is thermodynamically consistent, coupling hydrodynamics, surface tensions as well as bending stiffness, inextensibility and line tension of the membrane. We demonstrate the capabilities of our model through simulation results, which we compare to analytical solutions, alternative numerical approaches, and relevant biological processes.

References:

[1] https://www.sciencedirect.com/science/article/pii/S0045782526002112

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Formation of wrinkles on a coated substrate

Oliver Sander[1]

We present a numerical model for a large-strain hyperelastic object with part of the boundary coated by a Cosserat shell. When the stress-free configurations of the two objects do not match, wrinkles form, which is interesting for various applications. The numerical model combines geometric finite elements with standard finite elements, and we prove that its solutions exist. The resulting algebraic minimization problem is solved by Riemannian optimization, and we obtain a good match with experimental results.

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Notes on session “CFD”

Overlapping Schwarz Preconditioners for Pose Graph SLAM in Robotics

Yue Xiang Tee[1] Stephan Köhler[2] Oliver Rheinbach[3] Sebastian Zug[4]

We investigate scalable two-level overlapping Schwarz domain decomposition methods with energy-minimizing coarse spaces of GDSW type (Generalized Dryja–Smith–Widlund type) as preconditioners for the sparse linear systems arising in graph-based nonlinear least-squares problems, specifically the pose graph optimization back-end in Simultaneous Localization and Mapping (SLAM). After a brief introduction to SLAM and domain decomposition preconditioners, we describe the nonlinear least-squares formulation, its linearization, and the resulting matrix structure, to facilitate access for readers without prior knowledge of either field. Numerical experiments demonstrate the numerical scalability of the preconditioned conjugate gradient method (CG): Using the two-level overlapping Schwarz preconditioner, the number of CG iterations remains bounded independently of the problem size, overcoming the typical limitations of simple preconditioners, including one-level Schwarz approaches. We also consider two variants of Manhattan-world benchmark problems in SLAM.

References:

[1] https://doi.org/10.48550/arXiv.2603.08975

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Overlapping Schwarz Attention

Oliver Rheinbach[1] Stephan Köhler[2]

We investigate a hierarchical attention mechanism for use in Transformer architectures, based on additive two-level overlapping Schwarz domain decomposition. The construction replaces a single global attention interaction operator by local attention blocks on overlapping subdomains and a coarse attention level for long-range information exchange,

$$\Phi Q_0 K_0^T \Phi^T + \sum_{i=1}^{N} R_i^T D_i^{1/2} Q_i K_i^T D_i^{1/2} R_i,$$

where R_i restricts to an overlapping subdomain, D_i defines partition-of-unity weights, and Φ is a coarse interpolation matrix. We test the method in a linear operator-learning setting and compare it with global softmax-free low-rank attention.

Experiments with synthetic Fourier right-hand sides indicate faster convergence and higher accuracy than the global baseline, despite substantially fewer parameters.

References:

[1] https://arxiv.org/abs/2606.18525

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Surrogate-Assisted Global Sensitivity Analysis of Multilevel Block Preconditioners for Coupled Multiphysics Systems

Regina Bühler[1] Sebastian Brandstaeter[2] Alexander Popp[3] Matthias Mayr[4]

Finite element discretizations of coupled multiphysics problems yield large, sparse, block-structured linear systems. Solving these systems can account for a substantial share of the computational cost. Multilevel block preconditioners are one approach to solving such systems, but their performance depends on many parameters. Despite their potential influence on time-to-solution, these parameters are frequently selected based on experience or investigated only through limited parameter studies.

In this contribution, we investigate variance-based global sensitivity analysis as a systematic tool for identifying performance-critical solver parameters. Focusing on algebraic multigrid and block preconditioning, we treat solver parameters as uncertain inputs and time-to-solution as the quantity of interest. Sobol indices quantify the global influence of individual parameters and their interactions.

Since direct Monte Carlo estimation can involve many solver evaluations, we train a Gaussian process regression model on data obtained from solver runs and use it as a surrogate model to estimate the sensitivity indices inexpensively. Experiments on representative coupled multiphysics problems indicate that a subset of parameters has a dominant influence on solver performance, while others may be candidates for fixed default values.

References:

[1] A. Saltelli et al. Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index. Computer Physics Communications, 181(2):259–270, 2010.

[2] I. M. Sobol. Sensitivity estimates for nonlinear mathematical models. Math. Modelling Comput. Exp., 1(4):407–414, 1993

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Notes on session “Preconditioning”

On positivity preservation of HDG methods for the diffusion equation on hypergraphs

Petr Knobloch[1] Philip L. Lederer[2] Andreas Rupp[3]

Structure preserving discretizations of the diffusion equation need to be locally mass conservative and, in the case of nonnegative data, they should also preserve the positivity of the solution. This is particularly important in applications where the solution represents a physical quantity, such as temperature or concentration, which is nonnegative by definition. The most direct approach to obtain local mass conservation for the diffusion equation comprises dual, mixed methods, which discretize the flux explicitly. A way to bypass the saddle-point structure of the corresponding linear system lies in the concept of hybridization, which has been generalized to cover a large class of so-called hybrid discontinuous Galerkin (HDG) methods. These methods are locally mass conservative not only for classical domains but even if they discretize the diffusion equation posed on graphs or networks of surfaces, which naturally appear as singular limits of thin structures. The positivity preservation of such HDG methods has not been fully established, which is the focus of this contribution.

It will be shown that only a few low-order HDG methods on hypergraphs are positivity preserving. For nonobtuse simplicial meshes, this will be proved for the lowest-order Raviart–Thomas discretization with the stabilization parameter $\tau = 0$ and for piecewise constant approximations of all variables if $\tau = \mathcal{O}(1)$. For general meshes (also nonsimplicial ones), positivity preservation for piecewise constant approximations can be restored if $\tau = \mathcal{O}(1/h)$ is selected with h denoting the mesh width. This choice of τ leads to positivity preservation also for a few other discretizations. However, for most HDG methods, the positivity can be violated for any choice of τ as can be shown by appropriate counterexamples.

References:

[1] https://doi.org/10.1093/imanum/draf099

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High order biorthogonal functions for $H(div)$ on simplices

Sven Beuchler[1]

In this talk, we will present high order finite element discretizations using sparsity optimized basis functions. The aim of this contribution is to present biorthogonal functions to the above mentioned basis. We will discuss hexahedral and simplicial elements as well as situations on H1, H(div) and H(Curl). This work is a collaboration with T. Haubold (Paris) and J. Schoeberl (TU Vienna).

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An Anisotropic h-p Approach for Goal Oriented Adaptivity in Time and Space

Endtmayer Bernhard[1] Marius Paul Bruchhäuser[2] Nils Margenberg[3]

In this talk, we present an anisotropic goal-oriented error estimator based on the Dual Weighted Residual (DWR) method for a convection diffusion problem. We employ discontinuous elements in space and time. The directional error indicators quantify anisotropy of the solution with respect to a user defined goal functional, and produce hp-refinements that efficiently capture sharp layers. A numerical example demonstrate the superior performance of the proposed method compared to isotropic h and hp adaptive refinement.

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Notes on session “Discontinuous Galerkin”

Dynamical approximation and sensor placement for the state estimation of conservative dynamics

Cecilia Pagliantini[1]

This talk focuses on the inverse problem of reconstructing an unknown function from a finite set of measurements, under the assumption that such function is the output of a parametric evolution equation with unknown input parameters. In this talk we will discuss how to address this inverse problem for transport and wave phenomena by a reconstruction method that combines symplectic dynamical approximation with a dynamic strategy to position sensors. This talk is based on joint work with Olga Mula (University of Vienna) and Federico Vismara (RWTH Aachen).

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Modes of Mechanical Guidance of Adhesion-Independent Cell Migration

Hanna Luise Gertack[1] Sebastian Aland[2]

Adhesion-independent migration is a prominent mode of cell motility in confined environments, yet the physical principles that guide this movement remain incompletely understood. We present a phase-field model for deformable, non-adherent cells whose motility is driven by contractile cortical instabilities. The model couples surface and bulk hydrodynamics and includes a diffusible contractility regulator that drives cortical flows, making it well suited for complex geometries. Using this framework, we investigate how mechanical cues break symmetry to establish polarity and direct motion. We show that cells can spontaneously polarize and migrate in narrow channels, and that external cues – such as friction and viscosity gradients, geometric variations, imposed flows, and hydrodynamic interactions – can robustly steer migration. Beyond biological applications, this approach provides a generalizable phase-field methodology for modeling active surfaces, contractile instabilities, and hydrodynamic coupling.

References:

[1] https://doi.org/10.1039/d5sm00960j

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Elliptic PDEs on log-Gaussian Shapes: Sparsity and Finite Element Discretization

Helmut Harbrecht[1] Dinh Dung[2] Van Kien Nguyen[3] Christoph Schwab[4]

In this talk, we consider the solution to elliptic diffusion problems on a class of random domains obtained by log-Gaussian random homothety of the unit disk respectively an annulus. We model the problem under consideration and verify the existence and uniqueness of the random solution by path-wise pullback to the nominal unit disk respectively annulus. We prove the analytic regularity of the solution with respect to the random input parameter. We consider the numerical approximation of the random diffusion problem by means of continuous, piecewise linear Lagrangian Galerkin Finite Elements with numerical quadrature in the nominal domain, and by sparse grid interpolation and quadrature of Gauss-Hermite Smolyak and Quasi-Monte Carlo type in the parameter domain. The theoretical findings are complemented by numerical results.

References:

[1] https://arxiv.org/abs/2603.24056

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Notes on session “Applications”

Conditional Rectified Flow for stochastic homogenization of materials with random non-periodic structures

Hanyue Gu[1] Robert Gruhlke[2] Martin Eigel[3]

Analyzing properties of composite materials with random microstructures is of crucial importance in many engineering and industrial applications. Stochastic homogenization provides a framework to make the prohibitively high computational complexity tractable and for linking microscopic randomness to macroscopic behavior. Existing approaches typically focus on estimating the effective properties of individual microstructured materials, while characterization of their underlying distribution under varying conditions remains computationally challenging. We propose a conditional generative framework based on rectified flow to model the distribution of homogenized properties in random composites. Conditioning variables are incorporated into the training process, enabling efficient generation of effective properties for random domains with inclusions under prescribed conditions. The method is applied to stochastic diffusion and linear elasticity problems. Numerical results demonstrate that it accurately reproduces the target distributions of effective properties, providing an efficient data-driven surrogate for stochastic homogenization in heterogeneous media.

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Schwarz preconditioners for Discontinuous Galerkin method

Tomáš Hammerbauer[1]

We present a class of two level Schwarz preconditioners for the Discontinuous Galerkin (DG) method applied to second order elliptic problems. The preconditioners are based on a decomposition of the computational domain into non-overlapping subdomains and a coarse space that captures the global behavior of the solution. We analyze the convergence properties of the preconditioned system and demonstrate that the condition number is scalable with respect to the mesh size. Numerical experiments confirm the theoretical results and show that the proposed preconditioners are effective in reducing the number of iterations required for convergence, making them suitable for large-scale DG simulations.

References:

[1] https://arxiv.org/abs/2502.06405

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Notes on session “Poster Presentation”

Numerical challenges and techniques for higher order temporal discretizations applied to time-dependent generalized Stokes problems

Lukas Keller[1] Gunar Matthies[2]

When Galerkin time-stepping schemes and pressure-robust inf-sup stable finite element methods are applied to discretize time-dependent generalized Stokes equations, they yield a characteristic block-wise structure of the discretized system of equations. With increasing discretization order in time, these systems tend to be numerically ill-conditioned. We discuss strategies how to overcome that, especially by using a proper basis representation to calculate numerically precise coefficients of the time-stepping scheme. Additionally, we make use of the characteristic structure of the system matrix to reduce the computational effort. This is particular useful for higher order discretizations, where the system size tends to become quite large.

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General postprocessing for higher order hybrid temporal discretizations applied to transient Stokes problems

Gunar Matthies[1] Lukas Keller[2]

We consider a hybrid temporal discretization of the time-dependent generalized Stokes problem, using a continuous Galerkin-Petrov method for the velocity and a discontinuous approximation for the pressure. The spatial discretization is based on inf-sup stable finite element pairs for velocity and pressure. A cascadic interpolation of the right-hand side of the generalized Stokes problem allows to apply a sequence of postprocessing steps to successively improve the temporal behavior of both the velocity and the pressure. This process yields two improvements: a higher order convergence rate and higher global smoothness. Moreover, if the spatial discretization is pressure-robust, this property is transferred to the post-processed solution.

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A Space-Time Finite Element Method for the Stokes System on simplicial meshes

Tobias Kaltenbacher[1] Olaf Steinbach[2]

In this talk, we consider a space-time finite element method for the time-dependent Stokes system. While classical approaches rely on time-stepping schemes, we propose a fully space-time variational formulation in the Bochner setting. We first study the formulation on tensor-product space-time meshes and then extend it to completely unstructured meshes, allowing for a unified treatment of spatial and temporal discretization. Numerical experiments on arbitrary space-time meshes illustrate the flexibility and effectiveness of the proposed approach.

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Numerical Analysis of a Dirichlet Boundary Control Problem for the Stokes Equation

Katharina Lorenz[1] Thomas Apel[2]

In this talk, we consider a Dirichlet boundary control problem governed by the stationary Stokes equations with boundary controls in $L^2(\Gamma)^d$. The low regularity of the boundary control requires the use of a very weak formulation of the state equation. Within this framework, we derive the first-order optimality system for the control problem. Furthermore, we investigate the regularity of the adjoint variables, highlighting the limitations imposed by corner and edge singularities and identifying the maximal regularity that can be expected. The resulting regularity theory provides the analytical foundation for the finite element discretization and the derivation of a priori error estimates. Numerical experiments confirm the theoretical convergence rates.

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Notes on session “Stokes”

A flux-corrected ALE finite element scheme for porous-media conservation laws

Shahin Heydari[1] Dmitri Kuzmin[2] Thomas Wick[3]

We develop an arbitrary Lagrangian-Eulerian (ALE) flux-corrected transport (FCT) finite element method for conservation laws on moving meshes, with particular emphasis on transport in deformable porous media. The formulation is based on a conservative ALE weak form. Capacity-weighted nodal masses are evolved geometrically. Preservation of constant states is characterized by a discrete geometric conservation law (GCL). For homogeneous capacity, the fully discrete ALE scheme using Crank-Nicolson time stepping and the secant mesh velocity satisfies the GCL exactly. For variable capacity, we propose a divergence correction that synchronizes the evolution of geometric nodal masses with algebraic row sums of the discrete transport operator. A consistent-mass Galerkin update and a bound-preserving low-order predictor are combined in an element-based FCT algorithm. The antidiffusive term is assembled from element vectors with zero sums, which ensures discrete conservation. Spurious local extrema are prevented using an element-based version of Zalesak's limiter. Two-dimensional numerical examples illustrate the ability of the proposed approach to enforce geometric and algebraic constraints.

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Kernel-free BEM for variable coefficients

Benedikt Gräßle[1] Stefan A. Sauter[2]

Classical boundary element methods rely on explicit fundamental solutions and singular kernel evaluations, which are generally unavailable for variable-coefficient elliptic problems. We present a kernel-free framework that replaces the boundary operator with a computable algebraic approximation constructed in a one-time preprocessing step from a volume Galerkin discretisation. Coupling this approximation with boundary spaces yields a boundary-only system that retains the dimension reduction of classical BEM. Stability and quasi-optimal error estimates relate the boundary accuracy to the underlying volume approximation, while the resulting operator is compatible with data-sparse compression and efficient large-scale computation.

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Causality-Aware Training of Physics-Informed Neural Networks for the Wave Equation with Hard Constraints

Reza Mokhtari[1] Maryam Mohammadi[2]

While physics-informed neural networks (PINNs) have gained attention as flexible solvers for partial differential equations (PDEs), they often struggle to reliably approximate time-dependent dynamical systems exhibiting multi-scale or oscillatory behavior due to training instabilities and poor propagation of temporal information across time. Recent studies have highlighted the importance of enforcing temporal causality during training, demonstrating that appropriate weighting strategies can significantly improve convergence for evolutionary problems. In this talk, we investigate a causality-aware PINN framework for the following problem

$$\begin{cases} u_{tt}(x,y,t) - c^2\Delta u(x,y,t) = f(x,y,t), & (x,y) \in \Omega,\ t \in (0,T], \\ u(x,y,0) = \sin(k_1\pi x)\cos(k_2\pi y), & (x,y) \in \Omega, \\ u_t(x,y,0) = g(x,y), & (x,y) \in \Omega, \\ u(x,y,t) = 0, & (x,y) \in \partial\Omega,\ t \in [0,T], \end{cases}$$

where c denotes the wave velocity, Δ is the Laplacian operator, f is a given source term, $\Omega \subset \mathbb{R}^2$ is an open bounded domain, T is the final time, k_1 and k_2 are some constants in $\mathbb{R}$, and g is a given initial function. To improve training stability and physical consistency, we impose hard constraints to exactly satisfy boundary conditions, thereby reducing the search space of admissible solutions. Building upon the causal training strategy, we propose a novel weighting function that modifies the exponential accumulation of past residuals. Unlike existing approaches that rely solely on cumulative residual histories, the proposed formulation introduces a refined weighting mechanism that better balances early- and late-time contributions. This mitigates excessive attenuation or amplification of gradients and improves information propagation across time steps, resulting in enhanced convergence behavior. The proposed framework is compared with vanilla PINNs and the original causal weighting strategy. Numerical results show improved accuracy, faster convergence, and better long-time stability in capturing wave propagation. The effect of multiple hard constraints is also examined. Comparisons with the finite difference method (FDM) confirm the robustness and efficiency of the proposed approach for time-dependent wave problems.

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ML-Enhanced SUPG Stabilization for General Meshes

Manoj Prakash[1] Petr Knobloch[2]

Numerical simulation of fluid flows often requires solving convection–diffusion–reaction (CDR) equations, which model the transport of quantities such as temperature, concentration, or vorticity. In convection-dominated regimes, standard finite element methods suffer from spurious oscillations and instability. The Streamline Upwind/Petrov–Galerkin (SUPG) method addresses these issues by introducing a residual-based stabilization term controlled by an element-wise parameter δ_K. Since the choice of δ_K strongly influences solution accuracy, selecting an appropriate value remains a key challenge because classical parameter selection strategies often lead to either insufficient stabilization or excessive numerical diffusion.

In this work, we extend our machine learning–based framework for computing optimal SUPG stabilization parameters to unstructured and anisotropic meshes, which are far more common in practical CFD applications than structured meshes. A neural network is trained within the SUPG framework to learn local flow and mesh features and predict element-wise optimal stabilization parameters δ_K. The model is trained on a diverse range of mesh configurations, enabling robust generalization across structured, unstructured, and strongly anisotropic meshes. Comparative numerical experiments show that the proposed ML-enhanced SUPG method consistently produces more accurate solutions than the standard SUPG method on all tested mesh types, effectively suppressing spurious oscillations while maintaining the solution quality achieved on structured grids. These results demonstrate the potential of data-driven stabilization strategies for delivering reliable and accurate finite element solutions to convection-dominated problems on general computational meshes.

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Notes on session “Numerical Analysis I”

Numerical Modeling of Flow, Energy Transfer, and Deformation in Fractured Geothermal Systems

Inga Berre[1]

Engineered geothermal reservoirs exhibit strongly coupled thermo–hydro–mechanical–chemical (THMC) processes that govern reservoir evolution and performance during injection and production. For example, frictional sliding and opening of fractures and/or phase change arise from process-structure interactions among fluid flow, heat transport, rock deformation, stress transfer, and fracture geometries, representing strongly nonlinear multiphysics behaviour.

The presentation focuses on mathematical models and numerical methods for simulating the coupled multiphysics. The mathematical modeling is based on a mixed-dimensional approach, where fractures are modelled as co-dimension one objects. The surrounding rock is considered as thermo-poroelastic and the deformation of the fractures is modelled by frictional contact mechanics. We consider flow models of different complexity, ranging from single-phase isothermal models to thermal compositional models with phase separation. Various numerical challenges are highlighted and discussed, with emphasis on choice of model formulations and solvers. Numerical examples demonstrate how the proposed models can improve understanding of reservoir response to engineering operations.

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Phase-Field Modeling of Complex Interfaces: From Elastic Membranes to Energy-Stable Models for Viscous Surfaces

Maximilian Kloppe[1] Sebastian Aland[2]

The small thickness of membranes, shells, and capsules enables their efficient approximation as hypersurfaces. Phase-field modeling provides a versatile framework for modeling the motion of such complex interfaces in fluid flow. In the first part of this talk, we present our previously developed model for elastic and viscoelastic hypersurfaces in Navier–Stokes fluids. By coupling a phase-field approach for two-phase flow to an Eulerian description of the surface deformation tensor, we incorporate bending, tension, and in-plane stretching. We briefly highlight the versatility of this framework by demonstrating its application to the high-throughput mechanical characterization of lipid vesicles under flow.

However, a wide range of multiphase systems—ranging from dynamic emulsions and foams to surfactant-laden droplets—is governed by complex viscous surface rheology rather than in-plane elasticity. Capturing these phenomena requires a different rheological model that accounts for surface shear and dilatational viscosities. To address this need, the second part of this talk introduces a phase-field model tailored to viscous surfaces.

Coupled to the Navier–Stokes equations, our new formulation incorporates viscous surface stresses based on the classical Boussinesq–Scriven constitutive relation. Ensuring thermodynamic consistency and designing a corresponding structure-preserving discretization represent major mathematical and algorithmic challenges. We address this by developing an energy-stable mathematical framework that satisfies a continuous energy dissipation law. Finally, we discuss a robust discretization that preserves this energy-stable structure at the discrete level and present representative numerical examples of viscous interfaces in complex flows.

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A positivity preserving finite element scheme for a Keller–Segel system

Christos Pervolianakis[1] Panagiotis Chatzipantelidis[2]

In this talk, we consider the Keller–Segel system for chemotaxis on a bounded two-dimensional domain. The system consists of two coupled parabolic partial differential equations that describe the cell density and the chemical concentration. Since the standard finite element method fails to preserve the positivity of the discrete solutions, we consider a stabilized finite element scheme that preserves the positivity. This stabilized scheme is constructed using the Algebraic Flux Correction (AFC) method.

As the resulting fully discrete scheme is nonlinear, we discuss its well-posedness. In addition, we derive a residual-type a posteriori error estimator. Finally, we present numerical experiments that validate the theoretical results.

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Notes on session “Modelling”

Efficient Mapping of Objects to FEM Mesh for Bayesian Imaging

Colin Fox[1]

Bayesian imaging using a high-level representation interprets measured data in terms of an unknown number of 'objects', to infer the number and location of 'objects' that make up the system being imaged. When applied to inverse problems where data simulation requires solving a PDE, a significant computational cost can come from the mapping of objects to the FE mesh – assuming that we avoid the cost of remeshing by using a fixed mesh. We show a cheap but provably convergent scheme for that mapping, in the context of locating an unknown number of circular inclusions in electrical impedance tomography (EIT).

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Nested Stochastic and Geometric Multigrid for Matrix-Free Space-Time Stochastic Galerkin Systems

Moataz Dawor[1] Prof. Dr. Markus Bause[2]

Stochastic Galerkin finite element methods can exploit smooth dependence on moderate-dimensional random inputs, but stochastic enrichment produces large and strongly coupled algebraic systems. We consider random parabolic problems discretized by conforming finite elements in space, discontinuous Galerkin methods in time, and Hermite generalized polynomial chaos in the stochastic variables. The resulting slab systems are applied matrix-free and solved by right-preconditioned FGMRES.

The main focus is a nested stochastic multigrid-geometric multigrid (SMG-GMG) preconditioner and its comparison with a block-Jacobi-GMG (BJ-GMG) baseline. BJ-GMG treats the diagonal stochastic blocks independently using deterministic space-time geometric multigrid, leaving the off-diagonal stochastic couplings to the outer Krylov iteration. SMG-GMG additionally resolves these couplings through a hierarchy in the total polynomial degree, forward and backward block Gauss-Seidel smoothing in stochastic mode space, and deterministic geometric multigrid for the individual block corrections.

For benchmarks with four Gaussian variables, (Q_3) finite elements in space, and (dG(3)) discretization in time, the average FGMRES iteration count for SMG-GMG remains nearly independent of the stochastic polynomial degree, increasing only from approximately (6.2) to (7.3). In contrast, the iteration count for BJ-GMG increases from approximately (9.6) to (30.7). At the higher stochastic degrees, SMG-GMG therefore requires about four times fewer outer iterations and exhibits no iteration growth under the investigated space-time refinements. Since an SMG-GMG cycle is more expensive than one BJ-GMG application, however, the resulting wall-time improvement is presently modest.

Finally, the solver framework is illustrated for contaminant transport driven by stationary Darcy flow with lognormal hydraulic conductivity. The experiments demonstrate improved robustness with respect to stochastic enrichment and identify the cost and parallelization of the stochastic smoother as the principal remaining challenges for scalable stochastic Galerkin simulations of more complex multiphysical systems such as porous-media problems.

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Multi-Fidelity Monte Carlo Methods for Gas Dispersion Problems under Uncertain Inflow Conditions

Lisa Kühn[1] Jacopo Bonari[2] Max von Danwitz[3] Alexander Popp[4]

In this work, the dispersion of airborne contaminant is described by employing steady-state incompressible Navier–Stokes (INS) equations and a subsequent advection-diffusion (AD) problem. Varying surrounding wind conditions are additionally incorporated by a parametric Dirichlet boundary condition applied to the INS problem. To enable near real-time evaluations and computationally feasible multi-query analyses, previous works focused on deriving reduced-order model (ROM) approximations for the INS problem as the computational bottleneck in [1] and the coupled problem in [2]. These ROMs can replace the full-order model (FOM) in, for instance, Monte Carlo simulations. However, errors introduced by the ROM in approximating the wind field can lead to increased errors in the spatio-temporal concentration field, which accumulate over time. Therefore, Multi-Fidelity Monte Carlo [2] approaches are discussed to enhance the accuracy in Monte Carlo estimations while preserving computational efficiency. They rely on the developed ROM as low-fidelity model and the FOM as high-fidelity model. Furthermore, the relationship between the models is learned using Gaussian processes. By incorporating this relationship, Monte Carlo estimates are obtained, that closely approximate the high-fidelity results, with evaluations predominantly performed on the computationally efficient ROM. Thus, run times can be drastically reduced with only minor accuracy losses. This facilitates assessments of the influence of uncertain surrounding wind conditions and "what-if" studies for the presented application.

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Notes on session “Uncertainty”

Construction of trace-preserving Fortin operators

Franziska Eickmann[1] Tabea Tscherpel[2]

In the theory of mixed finite element spaces for incompressible fluid flows a so-called Fortin operator ensures that the inf-sup stability holds. It is a bounded (linear) operator that preserves the discrete divergence in the dual of the pressure space. Additional properties can be obtained by constructive means, for example locality or trace preservation properties. Those are key in the numerical analysis of non-Newtonian fluid flow and quasi-optimal error estimate for inhomogeneous Dirichlet boundary conditions. Also, when approximating curved domains they yield uniform inf-sup constants. We present a strategy to construct such Fortin operators for several H^1 conforming finite elements. This is joint work with Tabea Tscherpel.

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Exactly divergence-free finite element functions and boundary approximation

Tabea Tscherpel[1] L. Ridgway Scott[2]

Exactly divergence-free finite element functions allow for pressure-robust approximation of solutions to incompressible fluid equations. Approximating curved domains by polygons and imposing homogeneous Dirichlet boundary conditions exactly on the discrete boundary may introduce artificial constraints. These constraints reduce the approximation order.

We analyse these constraints for several finite element spaces and show how they can be avoided.

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Numerical treatment of Dirichlet boundary conditions in the finite element method

Thomas Apel[1]

For problems with Dirichlet boundary conditions, the boundary data are projected into a finite element space. There are various methods for doing this:

- L^2 projection,
- $H^{1/2}$ projection,
- Lagrange interpolation,
- Scott–Zhang interpolation,
- Carstensen interpolation,

which will be compared in the lecture. Variations arise when the data must satisfy compatibility conditions or when, as in the case of Dirichlet boundary control, the projector is of theoretical interest only but does not need to be implemented.

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Notes on session “Numerical Analsis II”

Curved Boundaries in Finite Elements

Rigdway Scott[1]

We review a number of techniques for dealing with curved boundaries involving polygonal approximation of the boundaries and finite elements.

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Maximum-Norm Stability of Isoparametric Finite Element Methods for Elliptic and Parabolic Equations in Curvilinear Polyhedra

Yupei Xie[1] Buyang Li[2] Weifeng Qiu[3] Wenshan Yu[4]

In this talk, I will present two published works from my PhD research at the Hong Kong Polytechnic University concerning the maximum-norm stability of isoparametric finite element methods on curvilinear polyhedral domains. In this setting, the domain cannot be triangulated exactly, and one has to deal with the discrepancy between the computational domain Ω_h and the exact domain Ω. Our results show, roughly speaking, that maximum-norm stability properties known for standard finite element methods without such domain discrepancy can be extended to the isoparametric setting.

For the Poisson equation with Dirichlet boundary conditions in a possibly concave curvilinear polyhedral domain whose edge opening angles are smaller than π, we extend the weak discrete maximum principle for standard Lagrange finite elements to the isoparametric finite element method. As an application, we prove a maximum-norm best-approximation property for the isoparametric finite element solution of the Poisson equation in a curvilinear polyhedron.

For the heat equation with homogeneous Dirichlet boundary conditions in a curvilinear polyhedral domain whose edge opening angles may be greater than π, we extend the analyticity and maximal regularity of the discrete semigroup, previously established for the standard semi-discrete Lagrange finite element method, to the isoparametric setting. As an application, we derive a quasi-optimal maximum-norm error estimate for the semi-discrete isoparametric finite element method.

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Isogeometric analysis on domains with curved boundaries and polar corners

Philipp Zilk[1]

Corner singularities pose a major obstacle to the efficient numerical solution of partial differential equations (PDEs). Their treatment is particularly delicate in isogeometric analysis (IGA) due to the tensor-product structure of classical spline spaces and the resulting need for advanced constructions that support local mesh refinement. At the same time, through the use of rational spline functions, which are widely used in computer-aided design for geometric modeling, IGA provides a natural framework for the exact representation of many curved boundaries. In our earlier works [1,2], we showed that combining mesh grading with polar parameterizations provides a simple and effective strategy for local refinement toward polar corners while retaining a boundary-fitted representation of the computational domain.

Since a complete theoretical analysis of isogeometric methods based on polar mappings is highly nontrivial, the framework in [1,2] was intentionally restricted in the spirit of a proof of concept. In particular, the setting was limited to single-patch domains in two dimensions, solutions of second-order PDEs with a single corner singularity, and reduced spline smoothness for large corner angles.

This talk presents recent advances and extensions of the proposed method. We show how local refinement via polar parameterizations can be generalized to more complex settings, including multi-patch domains with curved boundaries and multiple corners, as well as three-dimensional geometries. We also discuss applications to higher-order PDEs and the associated need for smoother parameterizations and spline spaces. Numerical examples demonstrate the effectiveness of the extended constructions and the practical relevance of polar mappings for the accurate approximation of singular PDE solutions on domains with curved boundaries and corners.

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Notes on session “Curved Boundaries”

Laplace-Beltrami operators non-planar polygonal meshes

Antonia Stavemann[1] Roland Herzog[2]

Discrete Laplace-Beltrami operators are central to geometry processing, with applications ranging from mean-curvature estimation and smoothing to distance calculation and bending simulation. While many discretization approaches are well understood on triangulated or even planar polygonal meshes, extending them to non-planar polygonal faces remains challenging due to the non-constant normal field. This talk presents and compares several approaches to discretize the Laplacian on non-planar polygonal meshes from a geometry-processing perspective.

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Virtual element methods on non-planar polygons

Roland Herzog[1] Antonia Stavemann[2]

The virtual element method (VEM) was devised as an extension of the finite element method for meshes with polygonal or polyhedral cells of coinciding geometric and topological dimension. It can likewise be used for surface meshes embedded in 3D with planar polygons. In this talk, we present various approaches to extend the VEM to non-planar polygonal meshes and discuss their advantages and shortcomings.

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A ϕ-FEM approach for time-dependent domains with unfitted meshes

Johan Marguet[1] Michel Duprez[2] Alexei Lozinsky[3] Igor Voulis[4]

In this work, we propose an unfitted finite element scheme to approximate the solution of the heat equation on moving domains. We use the φ-FEM paradigm in which the computational domain is described implicitly by a level-set function φ. This function is incorporated into the variational formulation in order to ensure the boundary conditions, which allows the use of unfitted meshes in space. Such a strategy avoids the need for remeshing at each time step and makes it possible to handle complex geometrical evolutions. We introduce a fully discrete scheme that combines the φ-FEM spatial discretization with a standard discontinuous finite element method in time. Under suitable mesh regularity assumptions, we establish a priori error estimates in classical norms. Finally, we present several numerical experiments that confirm the theoretical convergence results and illustrate the robustness and accuracy of the proposed method.

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Notes on session “Novel Discretizations”

Robust subspace correction methods for primal plasticity

Ann-Sophie Schotte[1] Oliver Sander[2]

In small-strain plasticity, the underlying variational inequality becomes ill-posed in H_0^1 for the limit of vanishing hardening. In practice, this leads to a severe deterioration of the convergence rate of standard domain decomposition iterative solvers. Our goal is to develop a robust nonlinear preconditioner for this nonlinear limit problem. Inspired by nonlinear multigrid methods and nonlinear multilevel Schwarz methods, we discuss a coarse space correction that exploits the structure of the kernel of the limit problem. The coarse space will be designed to capture global couplings within the model that otherwise lead to the decline of the rate of convergence.

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Solving small-strain crystal plasticity without evaluating a return map

Yoshi Diepelt[1] Stefan Löhnert[2] Oliver Sander[3]

Solution methods of small-strain crystal plasticity commonly need to use an active set newton method to solve the constrained optimization problem of the widely used return map algorithm. We present the truncated nonsmooth multigrid newton method to solve a different formulation of small-strain crystal plasticity. Using methods of convex analysis, small-strain crystal plasticity is presented in its primal formulation. The resulting incremental optimization problem will be solved with the truncated nonsmooth multigrid newton method. We compare the primal formulation and algorithm to the formulation of the return map algorithm.

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Notes on session “Computational Mechanics”

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