A flux-corrected ALE finite element scheme for porous-media conservation laws

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We develop an arbitrary Lagrangian-Eulerian (ALE) flux-corrected transport (FCT) finite element method for conservation laws on moving meshes, with particular emphasis on transport in deformable porous media. The formulation is based on a conservative ALE weak form. Capacity-weighted nodal masses are evolved geometrically. Preservation of constant states is characterized by a discrete geometric conservation law (GCL). For homogeneous capacity, the fully discrete ALE scheme using Crank-Nicolson time stepping and the secant mesh velocity satisfies the GCL exactly. For variable capacity, we propose a divergence correction that synchronizes the evolution of geometric nodal masses with algebraic row sums of the discrete transport operator. A consistent-mass Galerkin update and a bound-preserving low-order predictor are combined in an element-based FCT algorithm. The antidiffusive term is assembled from element vectors with zero sums, which ensures discrete conservation. Spurious local extrema are prevented using an element-based version of Zalesak's limiter. Two-dimensional numerical examples illustrate the ability of the proposed approach to enforce geometric and algebraic constraints.

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