Phase-Field Modeling of Complex Interfaces: From Elastic Membranes to Energy-Stable Models for Viscous Surfaces

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The small thickness of membranes, shells, and capsules enables their efficient approximation as hypersurfaces. Phase-field modeling provides a versatile framework for modeling the motion of such complex interfaces in fluid flow. In the first part of this talk, we present our previously developed model for elastic and viscoelastic hypersurfaces in Navier–Stokes fluids. By coupling a phase-field approach for two-phase flow to an Eulerian description of the surface deformation tensor, we incorporate bending, tension, and in-plane stretching. We briefly highlight the versatility of this framework by demonstrating its application to the high-throughput mechanical characterization of lipid vesicles under flow.

However, a wide range of multiphase systems—ranging from dynamic emulsions and foams to surfactant-laden droplets—is governed by complex viscous surface rheology rather than in-plane elasticity. Capturing these phenomena requires a different rheological model that accounts for surface shear and dilatational viscosities. To address this need, the second part of this talk introduces a phase-field model tailored to viscous surfaces.

Coupled to the Navier–Stokes equations, our new formulation incorporates viscous surface stresses based on the classical Boussinesq–Scriven constitutive relation. Ensuring thermodynamic consistency and designing a corresponding structure-preserving discretization represent major mathematical and algorithmic challenges. We address this by developing an energy-stable mathematical framework that satisfies a continuous energy dissipation law. Finally, we discuss a robust discretization that preserves this energy-stable structure at the discrete level and present representative numerical examples of viscous interfaces in complex flows.

References:

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