Causality-Aware Training of Physics-Informed Neural Networks for the Wave Equation with Hard Constraints

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While physics-informed neural networks (PINNs) have gained attention as flexible solvers for partial differential equations (PDEs), they often struggle to reliably approximate time-dependent dynamical systems exhibiting multi-scale or oscillatory behavior due to training instabilities and poor propagation of temporal information across time. Recent studies have highlighted the importance of enforcing temporal causality during training, demonstrating that appropriate weighting strategies can significantly improve convergence for evolutionary problems. In this talk, we investigate a causality-aware PINN framework for the following problem

$$\begin{cases} u_{tt}(x,y,t) - c^2\Delta u(x,y,t) = f(x,y,t), & (x,y) \in \Omega,\ t \in (0,T], \\ u(x,y,0) = \sin(k_1\pi x)\cos(k_2\pi y), & (x,y) \in \Omega, \\ u_t(x,y,0) = g(x,y), & (x,y) \in \Omega, \\ u(x,y,t) = 0, & (x,y) \in \partial\Omega,\ t \in [0,T], \end{cases}$$

where c denotes the wave velocity, Δ is the Laplacian operator, f is a given source term, $\Omega \subset \mathbb{R}^2$ is an open bounded domain, T is the final time, k_1 and k_2 are some constants in $\mathbb{R}$, and g is a given initial function. To improve training stability and physical consistency, we impose hard constraints to exactly satisfy boundary conditions, thereby reducing the search space of admissible solutions. Building upon the causal training strategy, we propose a novel weighting function that modifies the exponential accumulation of past residuals. Unlike existing approaches that rely solely on cumulative residual histories, the proposed formulation introduces a refined weighting mechanism that better balances early- and late-time contributions. This mitigates excessive attenuation or amplification of gradients and improves information propagation across time steps, resulting in enhanced convergence behavior. The proposed framework is compared with vanilla PINNs and the original causal weighting strategy. Numerical results show improved accuracy, faster convergence, and better long-time stability in capturing wave propagation. The effect of multiple hard constraints is also examined. Comparisons with the finite difference method (FDM) confirm the robustness and efficiency of the proposed approach for time-dependent wave problems.

References:

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