Maximum-Norm Stability of Isoparametric Finite Element Methods for Elliptic and Parabolic Equations in Curvilinear Polyhedra

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In this talk, I will present two published works from my PhD research at the Hong Kong Polytechnic University concerning the maximum-norm stability of isoparametric finite element methods on curvilinear polyhedral domains. In this setting, the domain cannot be triangulated exactly, and one has to deal with the discrepancy between the computational domain Ω_h and the exact domain Ω. Our results show, roughly speaking, that maximum-norm stability properties known for standard finite element methods without such domain discrepancy can be extended to the isoparametric setting.

For the Poisson equation with Dirichlet boundary conditions in a possibly concave curvilinear polyhedral domain whose edge opening angles are smaller than π, we extend the weak discrete maximum principle for standard Lagrange finite elements to the isoparametric finite element method. As an application, we prove a maximum-norm best-approximation property for the isoparametric finite element solution of the Poisson equation in a curvilinear polyhedron.

For the heat equation with homogeneous Dirichlet boundary conditions in a curvilinear polyhedral domain whose edge opening angles may be greater than π, we extend the analyticity and maximal regularity of the discrete semigroup, previously established for the standard semi-discrete Lagrange finite element method, to the isoparametric setting. As an application, we derive a quasi-optimal maximum-norm error estimate for the semi-discrete isoparametric finite element method.

References:

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[2] Weifeng Qiu and Yupei Xie. Stability, analyticity and maximal regularity of semi-discrete isoparametric finite element solutions of parabolic equations in curvilinear polyhedra. Mathematics of Computation, 95(360), 2026. doi:10.1090/mcom/4201.

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