Isogeometric analysis on domains with curved boundaries and polar corners

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Corner singularities pose a major obstacle to the efficient numerical solution of partial differential equations (PDEs). Their treatment is particularly delicate in isogeometric analysis (IGA) due to the tensor-product structure of classical spline spaces and the resulting need for advanced constructions that support local mesh refinement. At the same time, through the use of rational spline functions, which are widely used in computer-aided design for geometric modeling, IGA provides a natural framework for the exact representation of many curved boundaries. In our earlier works [1,2], we showed that combining mesh grading with polar parameterizations provides a simple and effective strategy for local refinement toward polar corners while retaining a boundary-fitted representation of the computational domain.

Since a complete theoretical analysis of isogeometric methods based on polar mappings is highly nontrivial, the framework in [1,2] was intentionally restricted in the spirit of a proof of concept. In particular, the setting was limited to single-patch domains in two dimensions, solutions of second-order PDEs with a single corner singularity, and reduced spline smoothness for large corner angles.

This talk presents recent advances and extensions of the proposed method. We show how local refinement via polar parameterizations can be generalized to more complex settings, including multi-patch domains with curved boundaries and multiple corners, as well as three-dimensional geometries. We also discuss applications to higher-order PDEs and the associated need for smoother parameterizations and spline spaces. Numerical examples demonstrate the effectiveness of the extended constructions and the practical relevance of polar mappings for the accurate approximation of singular PDE solutions on domains with curved boundaries and corners.

References:

[1] T. Apel and P. Zilk, Error estimates and graded mesh refinement for isogeometric analysis in the vicinity of polar corners, Computer Methods in Applied Mechanics and Engineering, 452, Paper No. 118695, 2026. https://doi.org/10.1016/j.cma.2025.118695

[2] T. Apel and P. Zilk, Isogeometric analysis of the Laplace eigenvalue problem on circular sectors: Regularity properties and graded meshes, Computers & Mathematics with Applications, 175, Pages 236-254, 2024. https://doi.org/10.1016/j.camwa.2024.09.018

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