

Program

	Monday 18 Sep	Tuesday 19 Sep	Wednesday 20 Sep	Thursday 21 Sep	Friday 22 Sep
9:00	Lecture F. Voigtlaender	Lecture D. Nuyens	Lecture I. Curato	Lecture D. Kressner	Lecture K. Zygalakis
10:00	Break	Break	Break	Break	Break
10:30	Lecture D. Nuyens	Lecture F. Voigtlaender	Lecture K. Zygalakis	Lecture I. Curato	Lecture D. Kressner
11:30	Lunch	Lunch	Lunch	Lunch	Lunch
13:00	Session 1 F. Filbir	Session 3 H. Matt	Trip	Session 5 K. Pozharska	End
13:25	J. Bresch	P. Geuchen		A. Horst	
13:50	F. Schoppert	F. Bozorgnia		F. Bartel	
14:15	M. Rauscher	D.S. Darlington		A. Herremans	
14:40	Break	Break		Break	
15:15	Session 2 M. Moeller	Session 4 R. Kunsch		Session 6 A. Owis	
15:40	Y. Zhou	Posters (S1)		Posters (S2)	
16:05	O. Lednyts'kyy	Posters (S1)		Posters (S2)	
16:30	End	End		End	

Monday, 18 Sep		Wednesday, 20 Sep	
9:00	F. Voigtlaender From Nonlinear Approximation to Sampling	9:00	I. Curato Statistical Learning Theory
10:00	Break	10:00	Break
10:30	D. Nuyens Lattice Methods and Applications	10:30	K. Zygalakis Stochastic Differential Equations, Discrete Approximations, and Connections to Optimization and Sampling Algorithms
11:30	Lunch	11:30	Lunch
13:00	F. Filbir On Bernstein and Nikolskii Inequalities for Diffusion Polynomials	13:00	Trip
13:25	J. Bresch Denoising of Sphere- and SO(3)-valued Data by Relaxed Tikhonov Regularization	Thursday, 21 Sep	
13:50	F. Schoppert Edge Detection with Polynomial Frames on the Sphere	9:00	D. Kressner Randomized low-rank approximation in finite and infinite dimensions
14:15	M. Rauscher Reconstruction of the Signature Embedding for Sequential Data	10:00	Break
14:40	Break	10:30	I. Curato Statistical Learning Theory
15:15	M. Moeller Sampling numbers and best trigonometric m -term approximation in weighted Wiener spaces	11:30	Lunch
15:40	Y. Zhou Efficient least squares approximation and collocation methods using radial basis functions	13:00	K. Pozharska Sampling recovery in the uniform and other norms
16:05	O. Lednyts'kyi Overview of methods to reduce complexity in modeling of chemical reactions	13:25	A. Horst Exploring a posterior with Besov prior through sampling
Tuesday, 19 Sep		13:50	F. Bartel On the reconstruction of functions from values at subsampled quadrature points
9:00	D. Nuyens Lattice Methods and Applications	14:15	A. Herremans Enriched approximation using clustered poles
10:00	Break	14:40	Break
10:30	F. Voigtlaender Approximation Properties of Neural Networks	15:15	A. Owis Analytical Solution of the Linear Quadratic Regulator
11:30	Lunch	15:40	Poster Session 2
13:00	H. Matt Universal approximation with complex-valued deep narrow neural networks	Friday, 22 Sep	
13:25	P. Geuchen Optimal approximation of C^k -functions using shallow complex-valued neural networks	9:00	K. Zygalakis Stochastic Differential Equations, Discrete Approximations, and Connections to Optimization and Sampling Algorithms
13:50	F. Bozorgnia Graph Based Semi-supervised Learning Using Spatial Segregation Theory	10:00	Break
14:15	D.S. Darlington Developing Mathematical Techniques for Analyzing and Modeling Complex and High-Dimensional Data Sets arising from Machine Learning	10:30	D. Kressner Randomized low-rank approximation in finite and infinite dimensions
14:40	Break	11:30	Lunch
15:15	R. Kunsch Randomized Approximation of Vectors – Adaptive vs. Non-adaptive	13:00	End
15:40	Poster Session 1		

Invited Lectures

Imma Curato, Technische Universität Chemnitz

Statistical Learning Theory

Daniel Kressner, EPFL Lausanne

Randomized low-rank approximation in finite and infinite dimensions

Dirk Nuyens, KU Leuven

Lattice Methods and Applications

Felix Voigtlaender, KU Eichstätt-Ingolstadt

From Nonlinear Approximation to Sampling,
Approximation Properties of Neural Networks

Konstantinos Zygalakis, University of Edinburgh

Stochastic Differential Equations, Discrete Approximations, and Connections to Optimization and Sampling Algorithms

Abstracts of Contributed Talks

On the reconstruction of functions from values at subsampled quadrature points

Felix Bartel

Technische Universität Chemnitz

This talk is concerned with function reconstruction from samples. The sampling points used in several approaches are (1) structured points connected with fast algorithms or (2) unstructured points coming from, e.g., an initial random draw to achieve an improved information complexity. We connect both approaches and propose a subsampling of structured points in an offline step. In particular, we start with structured quadrature points (QMC), which provide stable L2 reconstruction properties. The subsampling procedure consists of a computationally inexpensive random step followed by a deterministic procedure to further reduce the number of points while keeping its information. In these points functions (belonging to a RKHS of bounded functions) will be sampled and reconstructed from whilst achieving state of the art error decay. We apply our general findings on the d -dimensional torus to subsample rank-1 lattices, where it is known that full rank-1 lattices loose half the optimal order of convergence (expressed in terms of the size of the lattice). In contrast to that, our subsampled version regains the optimal rate since many of the lattice points are not needed. Moreover, we utilize fast and memory efficient Fourier algorithms in order to compute the approximation. Numerical experiments in higher dimensions support our findings.

Graph Based Semi-supervised Learning Using Spatial Segregation Theory

Farid Bozorgnia

Instituto Superior Técnico, Portugal

In this talk, I address graph-based semi-supervised learning employing the theory of competitive-type systems of partial differential equations (PDEs) to classify data when only a few labels are available. First, we define a discrete counterpart over connected graphs by using a direct analogue of the corresponding competitive system. Then we focus on a slightly different model motivated by the recent numerical results on the spatial segregation of reaction-diffusion systems. In this case, we show that the model has a unique solution and propose a novel classification algorithm based on it. Finally, we present some numerical experiments showing the efficiency of the method.

Denoising of Sphere- and $SO(3)$ -valued Data by Relaxed Tikhonov Regularization

Jonas Bresch

Technische Universität Berlin

Manifold-valued signal- and image processing has received attention due to novel image acquisition techniques. Recently, Condat (IEEE Trans. Signal Proc.) [1] proposed a convex relaxation of the Tikhonov-regularized nonconvex problem for denoising circle-valued data. In this paper, we show, based on Schur complement arguments, that this variational model can be simplified while leading to the same solution. Our simplified model can be generalized to higher dimensional spheres and to $SO(3)$ -valued data, where we rely on the quaternion representation of the later one. Standard algorithms from convex analysis can be applied to solve the resulting convex minimization problem. As proof-of-the-concept, we use the alternating direction method of minimizers to demonstrate the denoising behavior of the proposed method.

Coauthors: Robert Beinert, Gabriele Steidl, Institute of Mathematics, Technische Universität Berlin, Straße des 17. Juni 136, 10623 Berlin, Germany, <http://tu.berlin/imageanalysis>

[1] Laurent Condat (2022), *Tikhonov regularization of Circle-Valued signals*, IEEE Transactions on Signal Processing, Vol. 70

Developing Mathematical Techniques for Analyzing and Modeling Complex and High-Dimensional Data Sets arising from Machine Learning

David S. Darlington

University of Liberia, Liberia

Machine learning algorithms have revolutionized various domains by enabling the analysis and modeling of complex and high-dimensional data sets. However, as the size and complexity of data sets continue to increase, traditional mathematical techniques face challenges in effectively extracting meaningful insights and building accurate models. This research paper aims to explore the development of novel mathematical techniques for analyzing and modeling such complex and high-dimensional data sets. We present an overview of existing mathematical approaches, identifying their limitations, and propose innovative methodologies that address these challenges. The paper emphasizes the need for advanced mathematical techniques to enhance the interpretability, scalability, and generalizability of machine learning models.

On Bernstein and Nikolskii Inequalities for Diffusion Polynomials

Frank Filbir

Helmholtz Munich / TU Munich

The Bernstein and Nikolskii inequalities are of fundamental importance in approximation theory and related fields. Bernstein inequalities for trigonometric polynomials are known for a long time, indeed it dates back to 1912 when S.N. Bernstein proved this inequality for norms of trigonometric polynomials. A Bernstein inequality bounds the norm of the k -th derivative of a trigonometric polynomial by the norm of the polynomials itself, viz.

$$\|P^{(k)}\|_p \leq n^k \|P\|_p,$$

where $1 \leq p \leq \infty$ and P is a trigonometric polynomial of degree n .

S.M.Nikolskii showed in 1951 an inequality which compares two different norms of a trigonometric polynomial. More precisely, if $1 \leq p \leq q \leq \infty$ then

$$\|P\|_q \leq c \|P\|_p,$$

where c is a constant depending on the degree of the polynomial and the parameters p and q .

As these inequalities are indispensable for getting approximation results we would like to get them in a much more general setting. In this talk we will demonstrate how to derive these inequalities in the general setting of diffusion polynomials which are substitutes for the trig. polynomials if we work on quite general manifolds.

The talk is based on results obtained in collaboration with Hrushikey Mhaskar (Claremont Graduate University, Los Angeles, USA).

Optimal approximation of C^k -functions using shallow complex-valued neural networks

Paul Geuchen

Katholische Universität Eichstätt - Ingolstadt

We prove a quantitative result for the approximation of functions of regularity C^k (in the sense of real variables) defined on the complex cube $\Omega_n := [-1, 1]^n + i[-1, 1]^n \subseteq \mathbb{C}^n$ using shallow complex-valued neural networks. Precisely, we consider neural networks with a single hidden layer and m neurons, i.e., networks of the form $z \mapsto \sum_{j=1}^m \sigma_j \cdot \phi(\rho_j^T z + b_j)$ and show that one can approximate every function in $C^k(\Omega_n; \mathbb{C})$ using a function of that form with error of the order $m^{-k/(2n)}$ as $m \rightarrow \infty$, provided that the activation function $\phi : \mathbb{C} \rightarrow \mathbb{C}$ is smooth but not polyharmonic on some non-empty open set. Furthermore, we show that the selection of the weights $\sigma_j, b_j \in \mathbb{C}$ and $\rho_j \in \mathbb{C}^n$ is continuous with respect to f and prove that the derived rate of approximation is optimal under this continuity assumption. We also discuss the optimality of the result for a possibly discontinuous choice of the weights. This is joint work with Felix Voigtlaender.

Enriched approximation using clustered poles

Astrid Herremans

KU Leuven

Often there is expert knowledge available on the behavior of the solution to a certain problem, such as the dominant singular or oscillatory characteristics. One way to incorporate this information in a numerical method is by switching to an enriched approximation set, consisting of a conventional basis augmented with known features of the function to be approximated. Computing an approximation in such sets generally requires solving ill-conditioned linear systems, yet accurate results can still be obtained under reasonable conditions when switching to a rectangular approximation method. One example is the lightning approximation scheme recently developed by Trefethen et al. The approximation set includes clustered simple poles to approximate branch point singularities at known locations. These types of approximation problems arise when solving PDEs on domains with corners, which led to the development of lightning solvers. A natural question to ask is whether clustered poles can also be used to approximate singularities in multiple dimensions.

Exploring a posterior with Besov prior through sampling

Andreas Horst

Technical University of Denmark

In Bayesian inverse problems, the prior distribution incorporates a priori information about the unknown quantity into the inverse problem. The choice of prior distribution can be crucial for the estimation of the unknown quantity. Besov priors is a family of wavelet-based priors that can promote both smooth and non-smooth function regularity. In this talk we study the uncertainty behavior of a posterior with Besov prior through sampling. We adapt a posterior with Besov prior to be applicable for the Randomize-Then-Optimize sampler which we use to explore how specific choices of Besov prior affects the posterior in illustrative inverse problems.

Randomized Approximation of Vectors – Adaptive vs. Non-adaptive*Robert J. Kunsch**Technische Universität Chemnitz*

We study the approximation of vectors via algorithms with access to evaluations of arbitrary linear functionals as information oracles within the framework of information-based complexity.

We provide new lower and upper bounds in the randomized setting and highlight new insights into the difference between adaptive und non-adaptive methods.

Overview of methods to reduce complexity in modeling of chemical reactions

Oleksandr Lednyts'kyi
Universität Greifswald

Chemical reaction processes are developed, analyzed, designed and controlled by modeling of relevant sets of chemical reactions. Three obstacles are known to hinder their use: 1) the size of reaction sets, 2) a number of uncertain or unknown values of reaction rates and 3) the presence of multiple time scales owing to the presence of fast and slow reactions. This causes difficulties in simulating complex nonlinear models and in estimating the sensitivity of simulations to parameter uncertainties. This and to approach a simulation, much simpler models are required for reducing complex ones. Indeed, a reduced model might gain insights into the reaction system's behavior based on the key features and components of the system. Model order reduction methods can be classified as lumping, sensitivity analysis and time-scale analysis. In this talk, basic chemical models are reduced and discussed according to the three methods described below. In lumping, the original composition variables are transformed into a lower dimensional vector by applying either a continuum or a discrete approach. These approaches are based on sets of physical properties or on the reactivity of the compounds, respectively. Being useful when only limited information is available, lumping is difficult to accomplish for large nonlinear reaction systems. In sensitivity analysis, only significant reactions and species are retained on the basis of their impact on designated important species. While evaluating the sensitivity matrices of the full model is required to ensure the accuracy of the reduced model, the sensitivity analysis is often used assuming the quasi-steady-state approximation. In time-scale analysis, fast and slow reactions are identified, and the fast-time-scale reactions and species are assumed to be at the quasi-steady-state approximation, whereas the slow-time-scale reactions are subject to the corresponding constraints on the basis of their relationship to the fast-time-scale reactions. Mathematically formalized by singular perturbation theory, order reduction is achieved by replacing the differential equations for the fast species by algebraic relations. The applicability of the classic pseudo-steady-state hypothesis and partial equilibrium approximations is typically limited to simple reaction systems. Whereas the intuitive application of the quasi-steady-state approximations for the fast dynamics and its subsequent enforcement in the reduced-order model describing the slow dynamics is highly nontrivial in complex reaction systems.

Universal approximation with complex-valued deep narrow neural networks

Hannes Matt

Katholische Universität Eichstätt - Ingolstadt

We study the universality of complex-valued neural networks with bounded widths and arbitrary depths. Under mild assumptions, we give a full description of those activation functions $\varrho : \mathbb{C} \rightarrow \mathbb{C}$ that have the property that their associated networks are universal, i.e., are capable of approximating continuous functions to arbitrary accuracy on compact domains. Precisely, we show that deep narrow complex-valued networks are universal if and only if their activation function is neither holomorphic, nor antiholomorphic, nor \mathbb{R} -affine. This is a much larger class of functions than in the dual setting of arbitrary width and fixed depth. Unlike in the real case, the sufficient width differs significantly depending on the considered activation function. We show that a width of $2n + 2m + 5$ is always sufficient and that in general a width of $\max\{2n, 2m\}$ is necessary. We prove, however, that a width of $n + m + 4$ suffices for a rich subclass of the admissible activation functions. Here, n and m denote the input and output dimensions of the considered networks.

This is joint work with Paul Geuchen and Thomas Jahn.

Sampling numbers and best trigonometric m -term approximation in weighted Wiener spaces

Moritz Moeller

Technische Universität Chemnitz

The best m -term approximation has been a rather theoretical subject of study in approximation theory since its inception by Stechkin in 1955. Recently however Jahn, T. Ullrich and Voigtlaender have found some practical application for it by using it in a new bound on the sampling numbers. One important class of spaces where this bound can give an improvement over existing ones are weighted Wiener spaces. Motivated by this a new bound for the best m -term approximation in these spaces will be shown in this talk. This is achieved by using techniques from hyperbolic cross approximation. In addition a general optimality bound in form of a bound for the sampling numbers of these spaces will also be provided. Lastly, also tractable versions of these results will be presented.

Analytical Solution of the Linear Quadratic Regulator

Ashraf H. Owis

Cairo University, Egypt

In this work we will solve the problem of Linear Quadratic Regulator LQR in a closed form via solving the Matrix Differential Riccati Equation (MDRE). We introduce a new approach of solving MDRE via explicit matrix exponential algorithm, which requires only finding the eigenvalues of the Hamiltonian matrix. We exploit the properties of the eigenvalues of the Hamiltonian matrix to find a negative exponent solution. The resulting solution is general and independent of the initial state and the state of the system and therefore, is an analytical closed loop offline solution of the problem. The result is applied to a real world problem in astrodynamics.

Sampling recovery in the uniform and other norms

Kateryna Pozharska

Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine;

Faculty of Mathematics, Chemnitz University of Technology, Chemnitz, Germany

In the talk, recent results from [1] will be discussed on the recovery of functions in the uniform norm, as well as in more general seminorms, including L_p -approximation, based on function evaluations.

We obtain worst case error bounds for general classes of functions in terms of the best L_2 -approximation from a given nested sequence of subspaces combined with bounds on the Christoffel function of these subspaces. Besides an explicit bound, we obtain that there are linear algorithms using $2n$ samples that are as good as all algorithms using n arbitrary linear information up to a factor of at most \sqrt{n} ; a result that cannot be true without oversampling. Moreover, our results imply that linear sampling algorithms are optimal up to a constant factor for many reproducing kernel Hilbert spaces.

We consider also several applications of our theorems. These are L_p -approximation in

- several classes of periodic functions on the torus \mathbb{T}^d ,
- Sobolev spaces on manifolds, as well as
- approximation in L_∞ - and H^1 -norm for a weighted Sobolev space without uniformly bounded basis.

The advantage of our results is their generality and simplicity. The underlying (least squares) algorithm is based on a random construction and subsampling based on the solution of the Kadison-Singer problem. However, this result is non-constructive, but it shows the existence of an algorithm that is universal in p (for the approximation in the L_p -norm).

This is a joint work with David Krieg, Mario Ullrich and Tino Ullrich. KP would like to acknowledge support by the Philipp Schwartz Fellowship of the Alexander von Humboldt Foundation.

[1] D. Krieg, K. Pozharska, M. Ullrich, and T. Ullrich, *Sampling recovery in uniform and other norms*, arXiv:2305.07539v2, 2023.

Reconstruction of the Signature Embedding for Sequential Data

Marco Rauscher

Technische Universität München

In my talk I want to highlight signatures of sequential data. The signature is a mathematical tool to encode multidimensional processes or time series. It takes elements in the tensor space and consists of iterated integrals against the process. In this way, we can encode essential characteristics of the underlying process which can be used as features for machine learning models for example. Besides its useful properties, the space of signatures can be seen as a manifold. In this context, a natural question is if we can identify the shortest connection between two signatures within this manifold - namely the geodesics. In the second step, we will discuss the effects of perturbations of signatures on the underlying processes. Finally, I want to briefly highlight the effects of movements in the manifold along the geodesics on the corresponding processes. We plan to exploit these effects to identify efficient interpolation and sampling.

Edge Detection with Polynomial Frames on the Sphere

Frederic Schoppert

Universität zu Lübeck

The classical orthonormal basis of spherical harmonics provides a powerful tool for analyzing spherical signals. However, a big disadvantage is the fact that the information is only given in terms of global quantities. For this reason, one is often interested in localized polynomial frames to obtain position based information about the frequency content of a given signal. Our results affirm the efficiency of such systems, as we show that they are able to detect jump discontinuities which lie along smooth curves. Specifically, we present upper and lower estimates for the magnitude of the corresponding frame coefficients when the analysis function is concentrated in the vicinity of such a singularity.

Efficient least squares approximation and collocation methods using radial basis functions

Yiging Zhou

KU Leuven

We describe an efficient method for the approximation of functions using radial basis functions (RBFs), and extend this to a solver for boundary value problems on irregular domains. The method is based on RBFs with centers on a regular grid defined on a bounding box. This means that some of the centers are outside the computational domain, but otherwise exhibit structure. The equation is discretized using collocation with oversampling, with collocation points inside the domain only, resulting in a rectangular linear system to be solved in a least squares sense. The goal of this paper is the efficient solution of that rectangular system. We show that the least squares problem splits into a regular part, which can be expedited with the FFT, and a low rank perturbation, which is treated separately with a direct solver. The rank of the perturbation is influenced by the irregular shape of the domain and by the weak enforcement of boundary conditions at points along the boundary. The solver extends the AZ algorithm which was previously proposed for function approximation involving frames and other redundant approximation spaces.

Poster Sessions

Session 1 - Tuesday

Learning Regularization Parameter-Maps for Variational Image Reconstruction using Deep NN and Algorithm Unrolling

Jonas Bresch, TU Berlin

Mathematics: Without This IMPOSSIBLE for IT Industry research

Samanta Debabrata, Rochester Institute of Technology, Kosovo

Marcinkiewicz Zygmund inequalities for scattered and random data on the q -sphere

Thomas Jahn, Katholische Universität Eichstätt-Ingolstadt

Fast and direct inverse nonequispaced Fourier transforms

Melanie Kircheis, Technische Universität Chemnitz

Suicide Ideation Prediction Using CNN-BiLSTM and BERT

Christianah T. Oyewale, University of Ibadan, Nigeria

Handling Imbalanced Medical Datasets: Review of a Decade of Research

Mabrouka Salmi, Tebessa, Algeria

Wasserstein gradient flows for the MMD and Moreau envelopes

Victor Stein, Technische Universität Berlin

Session 2 - Thursday**Fourier-ANOVA Methods for Machine Learning and High-Dimensional Approximation**

K. Akhalaya & F. Nestler, Technische Universität Chemnitz

Unsteady MHD Generalized Couette flow in an annuli

Sani Isa, Department of Mathematics and Statistics, Yobe State University, Nigeria

Exploring a posterior with Besov prior through sampling

Anafi Jafar, Federal University Birnin Kebbi, Nigeria

An automatic optical measurement system for geometry acquisition and quality monitoring of joined pieces

Niloofer Kashefpour, Technische Universität Chemnitz

Stochastic epidemic model

Tahar Khalifa, University of Sousse, Tunisia

Unsupervised Representation Learning in Multivariate Time Series with Simulated Data

Thabang Lebese, Université Clermont Auvergne, France

Biological fluid flows through a stenotic artery with turbulence

Nimra Muqaddass, Università degli Studi di Palermo, Italy

The reconstruction of parameters in diffusive processes as a model of measurement

Katja Tüting, Technische Universität Braunschweig