Program

	Monday 18 Sep	**Tuesday** 19 Sep	**Wednesday** 20 Sep	**Thursday** 21 Sep	**Friday** 22 Sep
9:00	Lecture F. Voigtlaender	Lecture D. Nuyens	Lecture I. Curato	Lecture D. Kressner	Lecture K. Zygalakis
10:00	Break	Break	Break	Break	Break
10:30	Lecture D. Nuyens	Lecture F. Voigtlaender	Lecture K. Zygalakis	Lecture I. Curato	Lecture D. Kressner
11:30	Lunch	Lunch	Lunch	Lunch	Lunch
	Session 1	Session 3	Trip	Session 5	End
13:00	F. Filbir	H. Matt		K. Pozharska	
13:25	J. Bresch	P. Geuchen		A. Horst	
13:50	F. Schoppert	P. Schröter		F. Bartel	
14:15	M. Rauscher			A. Herremans	
14:40	Break	Break		Break	
	Session 2	Session 4		Session 6	
15:15	M. Moeller	R. Kunsch		P. Mathé	
15:40	Y. Zhou	Posters (S1)		Posters (S2)	
16:05		Posters (S1)		Posters (S2)	
16:30	End	End		End	

	Monday, 18 Sep
9:00	**F. Voigtlaender** From Nonlinear Approximation to Sampling
10:00	**Break**
10:30	**D. Nuyens** Lattice Methods and Applications
11:30	**Lunch**
13:00	**F. Filbir** On Bernstein and Nikolskii Inequalities for Diffusion Polynomials
13:25	**J. Bresch** Denoising of Sphere- and SO(3)-valued Data by Relaxed Tikhonov Regularization
13:50	**F. Schoppert** Edge Detection with Polynomial Frames on the Sphere
14:15	**M. Rauscher** Reconstruction of the Signature Embedding for Sequential Data
14:40	**Break**
15:15	**M. Moeller** Tractable sampling numbers and best trigonometric m-term approximation in weighted Wiener spaces
15:40	**Y. Zhou** Efficient least squares approximation and collocation methods using radial basis functions

	Tuesday, 19 Sep
9:00	**D. Nuyens** Lattice Methods and Applications
10:00	**Break**
10:30	**F. Voigtlaender** Approximation Properties of Neural Networks
11:30	**Lunch**
13:00	**H. Matt** Universal approximation with complex-valued deep narrow neural networks
13:25	**P. Geuchen** Optimal approximation of C^k-functions using shallow complex-valued neural networks
13:50	**P. Schröter** Approximation with partially periodic basis functions in high dimensions
14:15	**Break**
15:15	**R. Kunsch** Randomized Approximation of Vectors – Adaptive vs. Non-adaptive
15:40	**Poster Session 1**

	Wednesday, 20 Sep
9:00	**I. Curato** Statistical Learning Theory
10:00	**Break**
10:30	**K. Zygalakis** Stochastic Differential Equations, Discrete Approximations, and Connections to Optimization and Sampling Algorithms
11:30	**Lunch**
13:00	**Trip**

	Thursday, 21 Sep
9:00	**D. Kressner** Randomized low-rank approximation in finite and infinite dimensions
10:00	**Break**
10:30	**I. Curato** Statistical Learning Theory
11:30	**Lunch**
13:00	**K. Pozharska** Sampling recovery in the uniform and other norms
13:25	**A. Horst** Exploring a posterior with Besov prior through sampling
13:50	**F. Bartel** On the reconstruction of functions from values at subsampled quadrature points
14:15	**A. Herremans** Enriched approximation using clustered poles
14:40	**Break**
15:15	**P. Mathé** Approximation by neural networks in Barron spaces
15:40	**Poster Session 2**

	Friday, 22 Sep
9:00	**K. Zygalakis** Stochastic Differential Equations, Discrete Approximations, and Connections to Optimization and Sampling Algorithms
10:00	**Break**
10:30	**D. Kressner** Randomized low-rank approximation in finite and infinite dimensions
11:30	**Lunch**
13:00	**End**

Invited Lectures

Imma Curato, Technische Universität Chemnitz

Statistical Learning Theory

Daniel Kressner, EPFL Lausanne

Randomized low-rank approximation in finite and infinite dimensions

Dirk Nuyens, KU Leuven

Lattice Methods and Applications

Felix Voigtlaender, KU Eichstätt-Ingolstadt

From Nonlinear Approximation to Sampling,
Approximation Properties of Neural Networks

Konstantinos Zygalakis, University of Edinburgh

Stochastic Differential Equations, Discrete Approximations, and Connections to Optimization and Sampling Algorithms

Abstracts of Contributed Talks

On the reconstruction of functions from values at subsampled quadrature points

Felix Bartel

Technische Universität Chemnitz

This talk is concerned with function reconstruction from samples. The sampling points used in several approaches are (1) structured points connected with fast algorithms or (2) unstructured points coming from, e.g., an initial random draw to achieve an improved information complexity. We connect both approaches and propose a subsampling of structured points in an offline step. In particular, we start with structured quadrature points (QMC), which provide stable L2 reconstruction properties. The subsampling procedure consists of a computationally inexpensive random step followed by a deterministic procedure to further reduce the number of points while keeping its information. In these points functions (belonging to a RKHS of bounded functions) will be sampled and reconstructed from whilst achieving state of the art error decay. We apply our general findings on the d-dimensional torus to subsample rank-1 lattices, where it is known that full rank-1 lattices loose half the optimal order of convergence (expressed in terms of the size of the lattice). In contrast to that, our subsampled version regains the optimal rate since many of the lattice points are not needed. Moreover, we utilize fast and memory efficient Fourier algorithms in order to compute the approximation. Numerical experiments in higher dimensions support our findings.

Denoising of Sphere- and SO(3)-valued Data by Relaxed Tikhonov Regularization

Jonas Bresch
Technische Universität Berlin

Manifold-valued signal- and image processing has received attention due to novel image acquisition techniques. Recently, Condat (IEEE Trans. Signal Proc.) [1] proposed a convex relaxation of the Tikhonov-regularized nonconvex problem for denoising circle-valued data. In this paper, we show, based on Schur complement arguments, that this variational model can be simplified while leading to the same solution. Our simplified model can be generalized to higher dimensional spheres and to SO(3)-valued data, where we rely on the quaternion representation of the later one. Standard algorithms from convex analysis can be applied to solve the resulting convex minimization problem. As proof-of-the-concept, we use the alternating direction method of minimizers to demonstrate the denoising behavior of the proposed method.

Coauthors: Robert Beinert, Gabriele Steidl, Institute of Mathematics, Technische Universität Berlin, Straße des 17. Juni 136, 10623 Berlin, Germany, http://tu.berlin/imageanalysis

[1] Laurent Condat (2022), *Tikhonov regularization of Circle-Valued signals*, IEEE Transactions on Signal Processing, Vol. 70

On Bernstein and Nikolskii Inequalities for Diffusion Polynomials

Frank Filbir
Helmholtz Munich / TU Munich

The Bernstein and Nikolskii inequalities are of fundamental importance in approximation theory and related fields. Bernstein inequalities for trigonometric polynomials are known for a long time, indeed it dates back to 1912 when S.N. Bernstein proved this inequality for norms of trigonometric polynomials. A Bernstein inequality bounds the norm of the k-th derivative of a trigonometric polynomial by the norm of the polynomials itself, viz.

$$\|P^{(k)}\|_p \leq n^k \, \|P\|_p,$$

where $1 \leq p \leq \infty$ and P is a trigonometric polynomial of degree n.

S.M.Nikolskii showed in 1951 an inequality which compares two different norms of a trigonometric polynomial. More precisely, if $1 \leq p \leq q \leq \infty$ then

$$\|P\|_q \leq c \, \|P\|_p,$$

where c is a constant depending on the degree of the polynomial and the parameters p and q.

As these inequalities are indispensable for getting approximation results we would like to get them in a much more general setting. In this talk we will demonstrate how to derive these inequalities in the general setting of diffusion polynomials which are substitutes for the trig. polynomials if we work on quite general manifolds.

The talk is based on results obtained in collaboration with Hrushikeh Mhaskar (Claremont Graduate University, Los Angeles, USA).

Optimal approximation of C^k-functions using shallow complex-valued neural networks

Paul Geuchen

Katholische Universität Eichstätt - Ingolstadt

We prove a quantitative result for the approximation of functions of regularity C^k (in the sense of real variables) defined on the complex cube $\Omega_n := [-1,1]^n + i[-1,1]^n \subseteq \mathbb{C}^n$ using shallow complex-valued neural networks. Precisely, we consider neural networks with a single hidden layer and m neurons, i.e., networks of the form $z \mapsto \sum_{j=1}^m \sigma_j \cdot \phi(\rho_j^T z + b_j)$ and show that one can approximate every function in $C^k(\Omega_n; \mathbb{C})$ using a function of that form with error of the order $m^{-k/(2n)}$ as $m \to \infty$, provided that the activation function $\phi : \mathbb{C} \to \mathbb{C}$ is smooth but not polyharmonic on some non-empty open set. Furthermore, we show that the selection of the weights $\sigma_j, b_j \in \mathbb{C}$ and $\rho_j \in \mathbb{C}^n$ is continuous with respect to f and prove that the derived rate of approximation is optimal under this continuity assumption. We also discuss the optimality of the result for a possibly discontinuous choice of the weights. This is joint work with Felix Voigtlaender.

Enriched approximation using clustered poles

Astrid Herremans
KU Leuven

Often there is expert knowledge available on the behavior of the solution to a certain problem, such as the dominant singular or oscillatory characteristics. One way to incorporate this information in a numerical method is by switching to an enriched approximation set, consisting of a conventional basis augmented with known features of the function to be approximated. Computing an approximation in such sets generally requires solving ill-conditioned linear systems, yet accurate results can still be obtained under reasonable conditions when switching to a rectangular approximation method. One example is the lightning approximation scheme recently developed by Trefethen et al. The approximation set includes clustered simple poles to approximate branch point singularities at known locations. These types of approximation problems arise when solving PDEs on domains with corners, which led to the development of lightning solvers. A natural question to ask is whether clustered poles can also be used to approximate singularities in multiple dimensions.

Exploring a posterior with Besov prior through sampling

Andreas Horst
Technical University of Denmark

In Bayesian inverse problems, the prior distribution incorporates a priori information about the unknown quantity into the inverse problem. The choice of prior distribution can be crucial for the estimation of the unknown quantity. Besov priors is a family of wavelet-based priors that can promote both smooth and non-smooth function regularity. In this talk we study the uncertainty behavior of a posterior with Besov prior through sampling. We adapt a posterior with Besov prior to be applicable for the Randomize-Then-Optimize sampler which we use to explore how specific choices of Besov prior affects the posterior in illustrative inverse problems.

Randomized Approximation of Vectors – Adaptive vs. Non-adaptive

Robert J. Kunsch
Technische Universität Chemnitz

We study the approximation of vectors via algorithms with access to evaluations of arbitrary linear functionals as information oracles within the framework of information-based complexity.

We provide new lower and upper bounds in the randomized setting and highlight new insights into the difference between adaptive und non-adaptive methods.

Universal approximation with complex-valued deep narrow neural networks

Hannes Matt
Katholische Universität Eichstätt - Ingolstadt

We study the universality of complex-valued neural networks with bounded widths and arbitrary depths. Under mild assumptions, we give a full description of those activation functions $\varrho : \mathbb{C} \to \mathbb{C}$ that have the property that their associated networks are universal, i.e., are capable of approximating continuous functions to arbitrary accuracy on compact domains. Precisely, we show that deep narrow complex-valued networks are universal if and only if their activation function is neither holomorphic, nor antiholomorphic, nor $\mathbb{R}$-affine. This is a much larger class of functions than in the dual setting of arbitrary width and fixed depth. Unlike in the real case, the sufficient width differs significantly depending on the considered activation function. We show that a width of $2n + 2m + 5$ is always sufficient and that in general a width of $\max\{2n, 2m\}$ is necessary. We prove, however, that a width of $n + m + 4$ suffices for a rich subclass of the admissible activation functions. Here, n and m denote the input and output dimensions of the considered networks.

This is joint work with Paul Geuchen and Thomas Jahn.

Tractable sampling numbers and best trigonometric m-term approximation in weighted Wiener spaces

Moritz Moeller
Technische Universität Chemnitz

The best m-term approximation has been a rather theoretical subject of study in approximation theory since its inception by Stechkin in 1955. Recently however Jahn, T. Ullrich and Voigtlaender have found some practical application for it by using it in a new bound on the sampling numbers. One important class of spaces where this bound can give an improvement over existing ones are weighted Wiener spaces. Motivated by this a new bound for the best m-term approximation in these spaces will be shown in this talk. This is achieved by using techniques from hyperbolic cross approximation. In addition a general optimality bound in form of a bound for the sampling numbers of these spaces will also be provided. Lastly, also tractable versions of these results will be presented.

Sampling recovery in the uniform and other norms

Kateryna Pozharska
Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine;
Faculty of Mathematics, Chemnitz University of Technology, Chemnitz, Germany

In the talk, recent results from [1] will be discussed on the recovery of functions in the uniform norm, as well as in more general seminorms, including L_p-approximation, based on function evaluations.

We obtain worst case error bounds for general classes of functions in terms of the best L_2-approximation from a given nested sequence of subspaces combined with bounds on the Christoffel function of these subspaces. Besides an explicit bound, we obtain that there are linear algorithms using $2n$ samples that are as good as all algorithms using n arbitrary linear information up to a factor of at most $\sqrt{n}$; a result that cannot be true without oversampling. Moreover, our results imply that linear sampling algorithms are optimal up to a constant factor for many reproducing kernel Hilbert spaces.

We consider also several applications of our theorems. These are L_p-approximation in

- several classes of periodic functions on the torus $\mathbb{T}^d$,
- Sobolev spaces on manifolds, as well as
- approximation in L_∞- and H^1-norm for a weigthed Soboolev space without uniformly bounded basis.

The advantage of our results is their generality and simplicity. The underlying (least squares) algorithm is based on a random construction and subsampling based on the solution of the Kadison-Singer problem. However, this result is non-constructive, but it shows the existence of an algorithm that is universal in p (for the approximation in the L_p-norm).

This is a joint work with David Krieg, Mario Ullrich and Tino Ullrich. KP would like to acknowledge support by the Philipp Schwartz Fellowship of the Alexander von Humboldt Foundation.

[1] D. Krieg, K. Pozharska, M. Ullrich, and T. Ullrich, *Sampling recovery in uniform and other norms*, arXiv:2305.07539v2, 2023.

Reconstruction of the Signature Embedding for Sequential Data

Marco Rauscher
Technische Universität München

In my talk I want to highlight signatures of sequential data. The signature is a mathematical tool to encode multidimensional processes or time series. It takes elements in the tensor space and consists of iterated integrals against the process. In this way, we can encode essential characteristics of the underlying process which can be used as features for machine learning models for example. Besides its useful properties, the space of signatures can be seen as a manifold. In this context, a natural question is if we can identify the shortest connection between two signatures within this manifold - namely the geodesics. In the second step, we will discuss the effects of perturbations of signatures on the un- derlying processes. Finally, I want to briefly highlight the effects of movements in the manifold along the geodesics on the corresponding processes. We plan to exploit these effects to identify efficient interpolation and sampling.

Edge Detection with Polynomial Frames on the Sphere

Frederic Schoppert
Universität zu Lübeck

The classical orthonormal basis of spherical harmonics provides a powerful tool for analyzing spherical signals. However, a big disadvantage is the fact that the information is only given in terms of global quantities. For this reason, one is often interested in localized polynomial frames to obtain position based information about the frequency content of a given signal. Our results affirm the efficiency of such systems, as we show that they are able to detect jump discontinuities which lie along smooth curves. Specifically, we present upper and lower estimates for the magnitude of the corresponding frame coefficients when the analysis function is concentrated in the vicinity of such a singularity.

Approximation with partially periodic basis functions in high dimensions

Pascal Schröter
Technische Universität Chemnitz

In this Talk we consider an orthonormal basis, generated by a tensor product of Fourier basis functions, half period cosine basis functions, and the Chebyshev basis functions. We deal with the approximation problem in high dimensions related to this basis and design a fast algorithm to multiply with the underlying matrix, consisting of rows of the non-equidistant Fourier matrix, the non-equidistant cosine matrix and the non-equidistant Chebyshev matrix, and its transposed. This leads us to an ANOVA (analysis of variance) decomposition for functions with partially periodic boundary conditions through using the Fourier basis in some dimensions and the half period cosine basis or the Chebyshev basis in others. We consider sensitivity analysis in this setting, in order to find an adapted basis for the underlying approximation problem. More precisely, we find the underlying index set of the multidimensional series expansion. Additionally, we test this ANOVA approximation with mixed basis at numerical experiments, and refer to the advantage of interpretable results.

References:

[1] Felix Bartel, Daniel Potts and Michael Schmischke. *Grouped Transformations and Regularization in High-Dimensional Explainable ANOVA Approximation.* SIAM J. Sci. Comput. 44.3 (2022), 1606–A1631. DOI: 10.1137/20M1374547.

[2] Michael Schmischke. *Interpretable Approximation of High-Dimensional Data based on the ANOVA Decomposition.* Dissertation, Universitätsverlag Chemnitz, 2022. ISBN: 978-3-96100-164-4.

Efficient least squares approximation and collocation methods using radial basis functions

Yiging Zhou
KU Leuven

We describe an efficient method for the approximation of functions using radial basis functions (RBFs), and extend this to a solver for boundary value problems on irregular domains. The method is based on RBFs with centers on a regular grid defined on a bounding box. This means that some of the centers are outside the computational domain, but otherwise exhibit structure. The equation is discretized using collocation with oversampling, with collocation points inside the domain only, resulting in a rectangular linear system to be solved in a least squares sense. The goal of this paper is the efficient solution of that rectangular system. We show that the least squares problem splits into a regular part, which can be expedited with the FFT, and a low rank perturbation, which is treated separately with a direct solver. The rank of the perturbation is influenced by the irregular shape of the domain and by the weak enforcement of boundary conditions at points along the boundary. The solver extends the AZ algorithm which was previously proposed for function approximation involving frames and other redundant approximation spaces.

Poster Sessions

Session 1 - Tuesday

Learning Regularization Parameter-Maps for Variational Image Reconstruction using Deep NN and Algorithm Unrolling
Jonas Bresch, TU Berlin

Mathematics: Without This .... IMPOSSIBLE for IT Industry research
Samanta Debabrata, Rochester Institute of Technology, Kosovo

Marcinkiewicz Zygmund inequalities for scattered and random data on the q-sphere
Thomas Jahn, Katholische Universität Eichstätt-Ingolstadt

Fast and direct inverse nonequispaced Fourier transforms
Melanie Kircheis, Technische Universität Chemnitz

Suicide Ideation Prediction Using CNN-BiLSTM and BERT
Christianah T. Oyewale, University of Ibadan, Nigeria

Handling Imbalanced Medical Datasets: Review of a Decade of Research
Mabrouka Salmi, Tebessa, Algeria

Wasserstein gradient flows for the MMD and Moreau envelopes
Victor Stein, Technische Universität Berlin

Session 2 - Thursday

Fourier-ANOVA Methods for Machine Learning and High-Dimensional Approximation
K. Akhalaya & F. Nestler, Technische Universität Chemnitz

Unsteady MHD Generalized Couette flow in an annuli
Sani Isa, Department of Mathematics and Statistics, Yobe State University, Nigeria

Exploring a posterior with Besov prior through sampling
Anafi Jafar, Federal University Birnin Kebbi, Nigeria

An automatic optical measurement system for geometry acquisition and quality monitoring of joined pieces
Niloofar Kashefpour, Technische Universität Chemnitz

Stochastic epidemic model
Tahar Khalifa, University of Sousse, Tunisia

Unsupervised Representation Learning in Multivariate Time Series with Simulated Data
Thabang Lebese, Université Clermont Auvergne, France

Biological fluid flows through a stenotic artery with turbulance
Nimra Muqaddass, Università degli Studi di Palermo, Italy

The reconstruction of parameters in diffusive processes as a model of measurement
Katja Tüting, Technische Universität Braunschweig