Program

	Monday 22 Sep	**Tuesday 23 Sep**	**Wednesday 24 Sep**	**Thursday 25 Sep**	**Friday 26 Sep**
9:00	Lecture M. Bachmayr	Lecture C. Poon	Lecture J. Siegel	Lecture A. Repetti	Lecture M. Schaller
10:00	Break	Break	Break	Break	Break
10:30	Lecture M. Schaller	Lecture A. Repetti	Lecture M. Bachmayr	Lecture C. Poon	Lecture J. Siegel
11:30	Lunch	Lunch	Lunch	Lunch	Lunch
 13:00 13:30 14:00	Session 1 M. Quellmalz N. Rux M. Kircheis	Session 3 N. Schneider B. Käßemodel K. Sadik	Trip	Session 5 A. Gilbert E. Mindlberger F. Bartel	End
14:30	Break	Break		Break	
 15:00 15:30 16:00	Session 2 R. Hielscher N. Nagel M. Moeller	Session 4 F. Filbir S. Saier L. Weidensager		Session 6 Posters Posters Posters	
16:30	End	End	Dinner[1]	End	

[1] Location and time to be announced!

	Monday, 22 Sep
9:00	**M. Bachmayr** High-Dimensional Approximations for Parametric and Random PDEs
10:00	**Break**
10:30	**M. Schaller** Data-Driven Methods in Control: Error Bounds and Guaranteed Stability
11:30	**Lunch**
13:00	**M. Quellmalz** Fast and Accurate Approximation of High-Dimensional Radial Kernels via Slicing
13:30	**N. Rux** Numerical methods for kernel slicing
14:00	**M. Kircheis** ANOVA approximation for control systems
14:30	**Break**
15:00	**R. Hielscher** Estimating the plane normal distribution from two dimensional surface intersections
15:30	**N. Nagel** Structural results for energy minimizing point sets on the torus
16:00	**M. Moeller** Instance optimal function recovery in Wiener spaces

	Tuesday, 23 Sep
9:00	**C. Poon** Inverse Optimal Transport
10:00	**Break**
10:30	**A. Repetti** Proximal Optimization and Deep Learning Methods in Inverse Imaging Problems
11:30	**Lunch**
13:00	**N. Schneider** Adaptive approximation of time-dependent functions with (dis)continuous anisotropic space-time finite elements
13:30	**B. Käßemodel** Quantum Integration in Tensor Product Besov Spaces
14:00	**K. Sadik** A Nonlinear Time Second-Order Problem Coupling Local and Nonlocal Diffusion Equations for Image Processing
14:30	**Break**
15:00	**F. Filbir** Phase Retrieval in Action: Image Reconstruction from Spectrogram Measurements
15:30	**S. Saier** Trigonometric Shearlets and Where to Find Them
16:00	**L. Weidensager** Periodic Sobolev-Besov regularity in terms of Chui-Wang wavelet coefficients

	Wednesday, 24 Sep
9:00	**J. Siegel** The Approximation Theory of Neural Networks
10:00	**Break**
10:30	**M. Bachmayr** High-Dimensional Approximations for Parametric and Random PDEs
11:30	**Lunch**
13:00	**Trip**

	Thursday, 25 Sep
9:00	**A. Repetti** Proximal Optimization and Deep Learning Methods in Inverse Imaging Problems
10:00	**Break**
10:30	**C. Poon** Inverse Optimal Transport
11:30	**Lunch**
13:00	**A. Gilbert** QMC for uncertainty quantification of tumour growth and treatment modelled by a semilinear parabolic PDE
13:30	**E. Mindlberger** Sparse grids vs. random points for high-dimensional polynomial approximation
14:00	**F. Bartel** Minimal Subsampled Rank-1 Lattices for Multivariate Approximation with Optimal Convergence Rate
14:30	**Break**
15:00	**Poster Session**

	Friday, 26 Sep
9:00	**M. Schaller** Data-Driven Methods in Control: Error Bounds and Guaranteed Stability
10:00	**Break**
10:30	**J. Siegel** The Approximation Theory of Neural Networks
11:30	**Lunch**
13:00	**End**

Invited Lectures

Markus Bachmayr, RWTH Aachen University

High-Dimensional Approximations for Parametric and Random PDEs

Clarice Poon, University of Warwick

Inverse Optimal Transport

Audrey Repetti, Heriot-Watt University Edinburgh

Proximal Optimization and Deep Learning Methods in Inverse Imaging Problems

Manuel Schaller, TU Chemnitz

Data-Driven Methods in Control: Error Bounds and Guaranteed Stability

Jonathan Siegel, Texas A&M University

The Approximation Theory of Neural Networks

Abstracts of Contributed Talks

Fast and Accurate Approximation of High-Dimensional Radial Kernels via Slicing

Michael Quellmalz
Technische Universität Berlin, Germany

The fast computation of large kernel sums arises as a subproblem in any kernel method, including kernel density estimation, electrostatic particle simulation or gradient flows. We approach the problem by slicing, which relies on projections to one-dimensional subspaces via an adjoint Radon transform combined with fast Fourier summation. Numerical examples demonstrate that our slicing approach outperforms existing methods like random Fourier features on standard test datasets.

Numerical Methods for Kernel Slicing

Nicolaj Rux
Technische Universität Chemnitz

Kernels often are used in machine learning to track interactions between particles. Computing the kernel sums naively scales quadratic with the amount of particles. Recently Fourier methods combined with slicing found a way to reduce the time complexity from quadratic to linear dependence in terms of the number of particles. As premise the kernel needs to be sliced and its Fourier coefficients must be known. We interpret the slicing relation as an inverse problem and provide two algorithms to recover the sliced kernel in terms of its Fourier series. We provide extensive numerical experiments to measure speed an accuracy of our two methods.

ANOVA approximation for control systems

Melanie Kircheis
Technische Universität Chemnitz

The ANOVA (ANalysis Of VAriance) decomposition is a powerful tool for analyzing high-dimensional functions as it separates a function additively in the form

$$f(\boldsymbol{x}) = f_\emptyset + \sum_{j=1}^{d} f_{\{j\}}(x_j) + \sum_{1\leq i<j\leq d}^{d} f_{\{i,j\}}(x_i, x_j) + \cdots + f_{[d]}(x_1, \ldots, x_d),$$

where each term depends only on a subset of variables x_j with $j \subseteq \{1, \ldots, d\}$. Since it is unknown a priori which of the ANOVA terms are relevant for the representation, the number of terms is equal to 2^d and therefore grows exponentially in the dimension d. To mitigate this curse of dimensionality, the ANOVA approximation provides a data-driven implementation by leveraging the sparsity of interactions often present in real-world data.

In this talk, we are interested in approximating control systems of the form

$$H(\boldsymbol{x}, \boldsymbol{u}) = F(\boldsymbol{x}) + G(\boldsymbol{x})\boldsymbol{u}$$

with $\boldsymbol{x} \in \mathbb{R}^d$ and $\boldsymbol{u} \in \mathbb{R}^m$, when F and G are of low-dimensional structure. Since in general F and G cannot be sampled separately, the ANOVA approximation cannot be employed straightaway for these two functions. Moreover, we also do not wish to perform a full ANOVA approximation for the function H as this would incorporate numerous terms involving nonexistent interactions of the components u_l of $\boldsymbol{u}$. Alternatively, we introduce a method which exploits the linearity in $\boldsymbol{u}$ to efficiently approximate the system from given samples.

Joint work with: Manuel Schaller, Daniel Potts and Karl Worthmann.

Estimating the plane normal distribution from two dimensional surface intersections

Ralf Hielscher
TU Bergakademie Freiberg

We consider the inverse problem of determining the distribution of plane normals in a three-dimensional volume based on their intersection with its surface. This problem is of interest in the analysis of grain boundary distributions in polycrystalline materials. We present a computational cheap approach that combines ideas from kernel density estimation and stereology. We analyse the asymptotic behaviour of the estimator and present optimal kernel functions.

Structural results for energy minimizing point sets on the torus

Nicolas Nagel
Technische Universität Chemnitz

Inspired by numerical results of Hinrichs and Oettershagen we investigate the structure of point sets minimizing the L2-discrepancy on the torus. Equivalently, these are point sets that minimize the worst case error of the quasi-Monte Carlo method for integration over a certain periodic Sobolev space. These two notions can be interpreted as energy minimization problems, similar to the famous Thomson problem. We give some results concerning the global optimality of certain point configurations for a large class of such energies.

Joint work with Dmitriy Bilyk and Ian Ruohoniemi from the University of Minnesota.

Instance optimal function recovery in Wiener spaces

Moritz Moeller
Technische Universität Chemnitz

In this talk we discuss non-linear sampling recovery of multivariate functions using techniques from compressed sensing. We show that square root Lasso (`rLasso`) with a particular choice of the regularization parameter $\lambda > 0$ as well as orthogonal matching pursuit (`OMP`) after sufficiently many iterations provide noise blind decoders which efficiently recover multivariate functions from random samples. In contrast to basis pursuit the decoders (`rLasso`) and (`OMP`) do not require any additional information on the width of the function class in L_∞ and lead to instance optimal recovery guarantees. In the second part we deal with the reverse problem and show that any instance optimal recovery operator for this setting must use at least the same number of points (up to some loglog factors).

Adaptive approximation of time-dependent functions with (dis)continuous anisotropic space-time finite elements

Nick Schneider
Friedrich-Alexander-Universität Erlangen-Nürnberg

We study the approximation of real-valued functions $f : [0, T] \times \Omega \to \mathbb{R}$ with different smoothness in time and space for $T \in (0, \infty)$ and a d-dimensional, polyhedral Lipschitz domain Ω.

Starting with a tensor product partition consisting of prisms, we apply a certain sequence of iterations of spacial and temporal bisections, our atomic refinement method, to refine the initial partition. On the refined partition, we approximate the given function f with (dis)continuous anisotropic finite elements.

The talk is based on joint work with Pedro Morin (Universidad Nacional del Litoral, Santa Fe) and Cornelia Schneider (Friedrich-Alexander-Universität Erlangen-Nürnberg, Erlangen).

Quantum Integration in Tensor Product Besov Spaces

Bernd Käßemodel
Technische Universität Chemnitz

We begin with a brief introduction to the basic concepts of quantum computing and quantum information-based complexity for multivariate integration and approximation problems in various smoothness classes. We then discuss characterizations of functions in tensor product Besov spaces (mixed smoothness) using the tensorized Faber-Cieselski basis with coefficients based on mixed iterated differences. Relying on such a decomposition we develop a quantum algorithm to establish bounds for the worst case quantum integration error for this function class.

A nonlinear time second order problem coupling local and nonlocal diffusion equations for image processing

Khadija Sadik

Est Essaouira, Cadi Ayyad University Marrakech, Morocco

The primary objective of image denoising is to extract important features such as noise from images. Many PDE-based image restoration techniques [3, 1] rely on local information. However, given the strong capability of nonlocal methods to preserve textures and fine details, numerous approaches based on nonlocal diffusion have been proposed [2, 4]. In this work, we investigate a second-order nonlinear model that combines both local and nonlocal diffusion for image processing. This hybrid approach, which merges second-order nonlinear equations with local and nonlocal mechanisms, allows a powerful and faster denoising process. We establish the existence and uniqueness of the proposed model and demonstrate its effectiveness through denoising experiments (Fig. 1), comparing the results with several state-of-the-art methods from the literature.

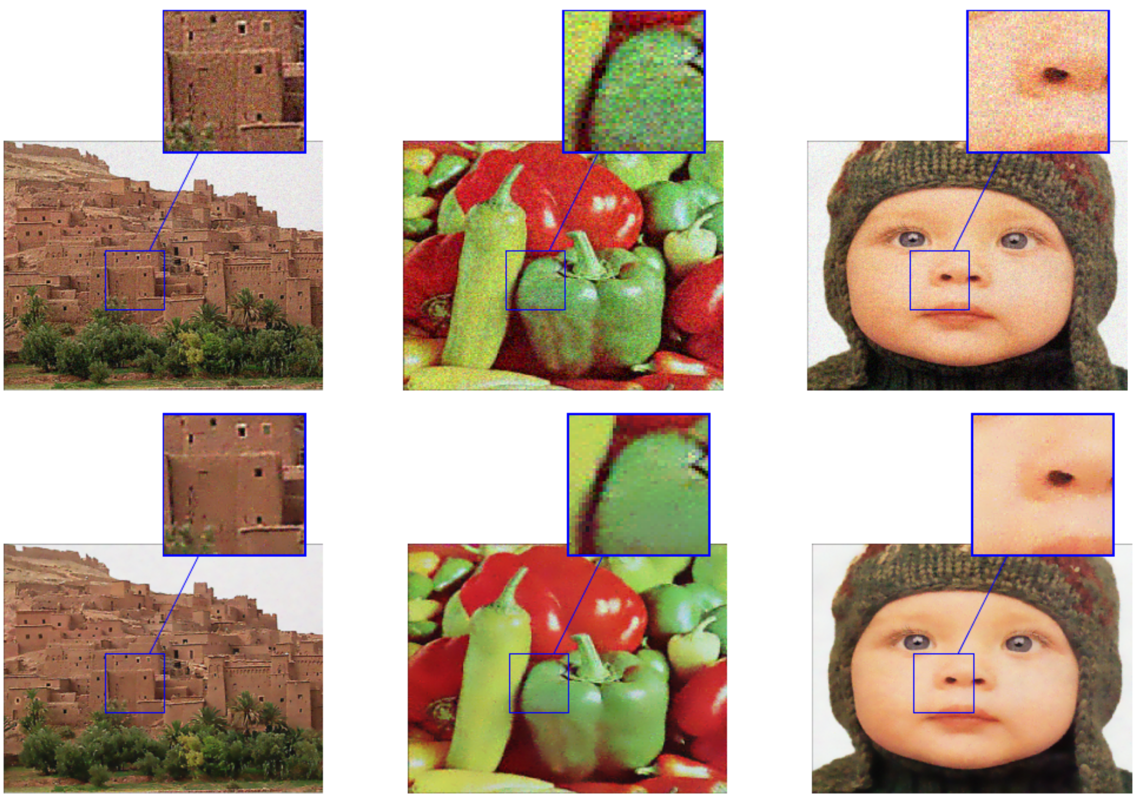

Figure 1: Denoising test of colored images using the proposed model. The first row shows noisy images, while the second row shows restored images.

Bibliography

[1] Francine Catté, Pierre-Louis Lions, Jean-Michel Morel, and Tomeu Coll. *Image selective smoothing and edge detection by nonlinear diffusion*. SIAM Journal on Numerical Analysis, 29(1):182–193, 1992.

[2] F. Karami, K. Sadik, and L. Ziad. *A variable exponent nonlocal $p(x)$-Laplacian equation for image restoration*. Computers and Mathematics with Applications, 75(2):534–546, 2018.

[3] Pietro Perona and Jitendra Malik. *Scale-space and edge detection using anisotropic diffusion*. IEEE Transactions on Pattern Analysis and Machine Intelligence, 12(7):629–639, 1990.

[4] Kehan Shi. *Image denoising by nonlinear nonlocal diffusion equations*. Journal of Computational and Applied Mathematics, 395:113605, 2021.

Phase Retrieval in Action
Image Reconstruction from Spectrogram Measurements

Frank Filbir
Helmholtz Munich & Technische Universität München

In ptychographic imaging a detector (CCD camera) measures the intensity of many diffraction patterns each obtained by illuminating a small part of the object at a time. The measurements are produced by using *X- rays* of one specific very short wavelength λ or an *electron beam*. The detector is usually placed in the far-field distance (Fraunhofer diffraction). Mathematically this experimental set-up leads to the problem of phase retrieval from spectrogram measurements. That means we are given samples of

$$\mathfrak{I}(x,\xi) := \Big| \int_{\mathbb{R}^2} f(t)\, g(t-x)\, \mathrm{e}^{-2\pi \mathrm{i}\, \xi\cdot t}\, \mathrm{d}t \Big|$$

for a known window function g. The aim is to reconstruct the object f. However, often experimental set-ups do not allow to work with one specific wavelength λ but we have to deal with polychromatic measurements, i.e. we are given

$$\mathfrak{I}_\lambda(x,\xi) = \Big| \int_{\mathbb{R}^2} f(t)\, g_\lambda(t-x)\, \mathrm{e}^{-2\pi \mathrm{i}\, \xi\cdot t/\lambda}\, \mathrm{d}t \Big|$$

for $\lambda \in \{\lambda_1, ..., \lambda_L\}$. Moreover, in many cases even the g_λ is unknown. This then leads to what is called *Blind Polychromatic Ptychographic Imaging* (BPPI). In this talk we will provide an overview of BPPI and we present some reconstruction methods and results.

Another important problem in material sciences is the reconstruction of the phase-gradient $\nabla\theta(t)$ of the function $f(t) = a(t)\,\mathrm{e}^{\mathrm{i}\,\theta(t)}$ from ptychographic measurements. We will comment briefly on the solution of this problem.

The talk is based on joint work with Oleh Melnyk (Technische Universität Berlin), and our project partners Jan Rothardt (GSI Jena) and Nico Hoffmann (HZDR, Dresden) within the Helmholtz Imaging Platform project AsoftXm, the group of Christian Schroer (DESY, Hamburg), and Xiaoke Mu (INT, KIT Karlsruhe).

Trigonometric Shearlets and Where to Find Them

Sofie Saier
Universität zu Lübeck

Edge detection is an important task in signal processing. But, while jump discontinuities in one-dimensional data are entirely characterized by their location, the formulation of corresponding higher-dimensional problems may also involve questions about local geometric properties of an edge.

Recent developments have introduced so-called shearlet systems as useful mathematical tools in the analysis of such questions. However, extensive research deals mainly with systems on the Euclidean spaces $\mathbb{R}^2$ or $\mathbb{R}^3$. We now consider a trigonometric shearlet system on the three-dimensional torus $\mathbb{T}^3$, in part even on higher-dimensional tori $\mathbb{T}^d$, and generalize localization results that were proven to be crucial in the context of two-dimensional singularity detection on $\mathbb{T}^2$. Therefore, we formalize, for the first time, directional localization of trigonometric functions in arbitrary dimensions and unveil the role of a certain type of smoothness in the frequency domain.

These results were part of my master's thesis supervised by Jürgen Prestin.

Periodic Sobolev-Besov regularity in terms of Chui-Wang wavelet coefficients

Laura Weidensager
Technische Universität Chemnitz

This paper investigates the representation of periodic Sobolev and Besov norms in terms of wavelet coefficients. Function spaces of mixed smoothness, fundamental in functional analysis and approximation theory, are traditionally defined through weak derivatives, integrability conditions, and smoothness parameters. By studying wavelet bases, we derive equivalent norms for these spaces expressed as weighted sums of wavelet coefficients, including explicit constants. This reveals the interplay between the function spaces and wavelet properties such as smoothness, vanishing moments, and scaling. These characterizations provide computational advantages and offer a unified perspective on Sobolev and Besov spaces, emphasizing their hierarchical structure and scale-dependent behavior.

QMC for uncertainty quantification of tumour growth and treatment modelled by a semilinear parabolic PDE

Alexander Gilbert
Univerity of New South Wales, Australia

TBA

Sparse grids vs. random points for high-dimensional polynomial approximation

Elias Mindlberger
Johannes Kepler University Linz, Austria

We study polynomial approximation on a d-cube, where d is large, and compare interpolation on sparse grids, aka Smolyak's algorithm (SA), with a simple least squares method based on randomly generated points (LS) using standard benchmark functions.

Our main motivation is the influential paper [Barthelmann, Novak, Ritter: High dimensional polynomial interpolation on sparse grids, Adv. Comput. Math. 12, 2000]. We repeat and extend their theoretical analysis and numerical experiments for SA and compare to LS in dimensions up to 100. Our extensive experiments demonstrate that LS, even with only slight oversampling, consistently matches the accuracy of SA in low dimensions. In high dimensions, however, LS shows clear superiority.

Minimal Subsampled Rank-1 Lattices for Multivariate Approximation with Optimal Convergence Rate

Felix Bartel[1], *Alexander D. Gilbert, Frances Y. Kuo, Ian H. Sloan*
[1]*Univerity of New South Wales, Australia*

In this talk we show error bounds for randomly subsampled rank-1 lattices. We pay particular attention to the ratio of the size of the subset to the size of the initial lattice, which is decisive for the computational complexity. In the special case of Korobov spaces, we achieve the optimal polynomial sampling complexity whilst having the smallest initial lattice possible. We further characterize the frequency index set for which a given lattice is reconstructing by using the reciprocal of the worst-case error achieved using the lattice in question. This connects existing approaches used in proving error bounds for lattices. We make detailed comments on the implementation and test different algorithms using the subsampled lattice in numerical experiments.