Convergence rates for bilinear and quadratic inverse problems based on tensorial liftings

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Considering the question: ‘How non-linear may a non-linear operator be in order to extend the linear regularization theory?’, we introduce the class of dilinear operators that embraces linear, bilinear, and quadratic mappings between Banach spaces. The set of dilinear operators covers many interesting applications arising in imaging and physics like blind deconvolution, deautoconvolution, or phase retrieval.

Based on the universal property of the tensor product, the central idea is to lift the non-linear mappings to linear representatives on a suitable topological tensor space. At the same time, we extend the class of usually convex regularization functionals to the class of diconvex functionals, which are likewise defined by a tensorial lifting. Owing to the lifting, we get immediate access to the linear regularization theory. On the downside, a simple lifting of the original inverse problem causes an additional non-convex rank-one constraint, which is as challenging to handle as the original non-linear problem. For this reason, most results of the linear regularization theory are not applicable for the lifted problem. To overcame this issue, we use the tensorial lifting implicitly and generalize the required concepts from convex analysis to the new framework. Generalizing subgradients and Bregman distances, we establish convergence rates under similar conditions as in the linear setting.

To demonstrate the applicability of the developed theory, we study the deautoconvolution problem in more detail. Although the unregularized problem can have at most two different solutions, the deautoconvolution problem is locally ill posed everywhere. Moreover, establishing theoretical convergence rates is very challenging since most conditions for the non-linear theory are not fulfilled. Applying the developed dilinear theory, we establish convergence rates under satisfiable source conditions.

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