The discrepancy principle for stochastically sampled data

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We consider the ill-posed linear equation $Kx = y$ in infinite dimensional Hilbert spaces. We assume that we have noisy but multiple measurements $y_1, ..., y_n$ of the true value y. Furthermore, assuming that the noisy measurements are unbiased and independently and identically distributed according to an unknown distribution, the natural approach would be to use $(y_1 + .. + y_n)/n$ as an approximation to y with the estimated error σ/n, where σ is an estimation of the standard deviation of one measurement. We show that this approach together with a deterministic regularisation method indeed yields convergence to the true solution x (in probability, but in gerneral not in L^2), when the number of measurements tends to infinity. Especially we show that using the discrepancy principle to determine the regularisation parameter leads, in a certain sense, to optimal convergence rates.

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